Countervailing power and input pricing: When is a waterbed effect likely?

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Abstract
A downstream firm with countervailing power can extract a reduced price from an input supplier. A waterbed effect occurs if this price reduction leads the input supplier to raise the price that it charges another downstream firm. Policy makers have been concerned that this waterbed effect could undermine downstream competition, and it was considered in detail in the 2008 UK grocery inquiry. This paper presents a simple but parsimonious model to investigate if and when a waterbed effect may arise. It shows that the effect may arise through optimal pricing behaviour, but that this critically depends on the nature of upstream technology, downstream competition and consumer demand. In particular, downstream competition tends to work against a waterbed effect, but convex upstream costs support the effect. The analysis is complementary to recent academic work on the waterbed effect that focuses on bargaining constraints.

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1 Introduction

A downstream firm has countervailing power if it can extract a price discount from an upstream supplier.\(^1\) There is a long literature considering how countervailing power can arise, particularly for large downstream firms.\(^2\) Recently, concerns have been raised about the effect of one downstream firm’s countervailing power on its downstream competitors. In particular, can countervailing power lead to a ‘waterbed effect’, where the discount achieved by one downstream firm leads the upstream supplier to raise prices to other downstream firms?\(^3\) Analysis of the waterbed effect was a key issue in the UK Competition Commission’s (2008) grocery inquiry.\(^4\)

This paper presents a simple but parsimonious model to analyse the relationship between countervailing power and downstream pricing. An upstream monopolist supplies an input to an arbitrary number of downstream firms. The supplier sets its prices to the downstream firms to maximize its profit, subject to any constraints on the price for individual firms. From an initial set of equilibrium prices, we consider the effects if the pricing constraint on one downstream firm tightens. If the supplier has to lower its supply price to that downstream firm, how does it alter the profit maximizing prices that it sets for all other downstream firms?

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\(^1\)See, for example, Galbraith (1952) and Snyder (2008).

\(^2\)For example, countervailing power may arise due to the existence of outside options, (Katz (1987) and Inderst and Wey (2003)), or, for a large buyer, due to the relative ‘importance’ of the buyer when the aggregate surplus function is concave (Chipty and Snyder (1999) and Inderst and Wey (2007)).

\(^3\)Countervailing power may also result in non-price effects, such as changing the incentives for an upstream supplier to invest in product or process innovation. See Inderst and Wey (2007).

\(^4\)In particular, see the discussion at paragraphs 5.19 to 5.43 and in appendix 5.4. Recent academic work on this issue, includes Chen (2003), Majumdar (2005), Inderst (2007), and Inderst and Valletti (2011).
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The model shows that any changes in downstream pricing involve three effects: a competition effect, a cost effect and an elasticity effect.

If one firm gains a lower input price this tends to undermine the competitive position of downstream rivals. From the supplier’s perspective, the demand from these rivals falls, lowering the profit maximizing prices it charges to these rivals. Thus downstream competitive interaction acts against a waterbed effect, but may assist a waterbed effect for downstream firms selling complementary products that use the common input.\(^5\)

Second, a common rationale for a waterbed effect relates to upstream production costs. If one downstream firm gains a cheaper supply price and buys more of the input from the supplier, then there is ‘less’ upstream supply for other downstream firms.\(^6\) The results presented below formalize this idea. A waterbed effect may arise when upstream production costs are strictly convex. In this situation, increased purchases by one downstream firm raises the marginal upstream supply cost for all other downstream firms. The profit maximizing supply prices set by the upstream firm tend to increase as marginal cost rises.\(^7\)

\(^5\)The literature has focussed on the relationship between pricing for downstream competitors. The model in this paper, however, is flexible enough to allow downstream firms to be competitors, complementors or independent.

\(^6\)The Competition Commission (2008, appendix 5.4, paragraph 48) notes that “40 percent of suppliers indicate that when demand from large customers increases, there could be supply shortages to small grocery retailers, . . . ”. That said, the cost-based argument is often stated in vague terms of ‘cost recovery’. Thus, the European Commission (2001) at paragraph 126 notes that a reduction in the input price to one downstream firm may lead an upstream supplier to ‘try to recover’ the price reduction from other downstream firms.

\(^7\)This cost effect, unlike, say, Chippy and Snyder (1999), is not related to bargaining or to the size of the downstream firm. Rather, it simply reflects that a downstream firm with a lower input price will buy more of the input and this has consequences for upstream marginal costs.
Third, if a fall in the supply price for one downstream firm significantly alters the elasticity of upstream demand for other downstream firms, this will change the prices that the supplier sets for these other downstream firms. For example, if a change in the input price for one downstream firm changes firm-specific downstream demand so that a competitor makes fewer sales but makes those sales to less-price-sensitive customers, then this can make the competitor’s upstream input demand less elastic. As a result, the profit maximizing supply price for this competitor can rise.

The model presented below is flexible. It is able to consider any number of downstream firms and a variety of downstream competitive or complementary relationships. That said, we focus attention of the two- and three-firm cases as these are more tractable. The model can also allow for multiple pricing constraints so that more than one downstream firm can have countervailing power and we analyse this in detail in the three-firm case.

A number of recent papers have considered the possibility of a waterbed effect through ‘linked’ constraints. For example, Inderst and Valleti (2011) show how a waterbed effect can arise when the bargaining positions of downstream firms are inter-related.\(^8\) Our model can also allow for ‘linked’ constraints in the sense that a tightening of the supply price constraint for one downstream firm can either tighten or slacken the constraint for another downstream firm. The model presented below can analyse how these link-

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\(^8\)More formally, downstream firms have access to an outside option at a fixed cost. If one downstream firm gains a discount on its input price then it lowers its retail price and gains a larger market share. With fewer customers, the outside option becomes a less credible alternative for the other downstream firm(s) and worsens its bargaining position with the upstream supplier. This results in the other downstream firm receiving a higher input price. This possibility was also considered informally in Competition Commission (2008). See also Inderst (2007). Chen (2003) also involves a strategic linkage. In his model the upstream firm can commit to prices for a competitive fringe and this ‘market price’ feeds into negotiations between the supplier and the dominant downstream firm.
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ages effect the input prices for third-firms. This is formally considered in section 5.3.

Our model provides significant guidance to policy makers. Like other recent models, a waterbed effect may arise if downstream firms’ bargaining positions (modelled here as pricing constraints) are linked. However, even in the absence of such linkage, a waterbed effect can arise, particularly if upstream costs rise at a rapid rate. Thus, for example, a waterbed effect is more likely if the upstream supplier faces capacity constraints. A waterbed effect may also arise - and indeed could be more likely to arise - where downstream firms are independent (or even sell complementary products). For example, if an upstream firm supplies an input that is used by downstream firms in geographically separate markets (e.g. different countries) then geographically distant firms may be more likely to face an input price rise than firms that are close competitors to the specific downstream firm that exploits increased countervailing power. Put simply, policy makers may have been looking for a waterbed effect in the wrong place!

2 The model

Consider a market where a single upstream supplier, denoted by $u$, supplies a homogeneous input to $m$ downstream firms. The $m$ downstream firms use the input to produce final goods and services and interact in the sale of these goods and services to final consumers. The upstream firm maximizes its profits by setting a vector of input prices $\mathbf{w} = (w_1, \ldots, w_m)$ where $w_i$ is the price that the upstream supplier charges the downstream firm $i$ to purchase its input.

Each downstream firm produces only one type of output and sets a price for that output of $p_i$ per unit, $i \in \{1, \ldots, m\}$. The outputs of the downstream firms can differ, however, for each downstream firm $i$, production of one unit
of output requires a fixed number of units of the input. Without loss of
generality, we can define the units of output for each firm \( i \) such that one
unit of input is required for one unit of output.\(^9\) We denote the production
of downstream firm \( i \) by \( q_i \) and, by our normalization, firm \( i \) requires \( q_i \) units
of the upstream input to produce the \( q_i \) units of final output.\(^10\)

Let \( p \) denote the vector of downstream prices \( (p_1, \ldots, p_m) \), \( p_{-i} \) denote the
vector of prices \( (p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_m) \) and \( \pi_i(p, w_i) \) denote the profit of
downstream firm \( i \) given downstream output prices and the price \( i \) pays for
the input. We assume that for any relevant set of input prices \( w \) there is a well
defined equilibrium price vector \( \tilde{p}(w) = (\tilde{p}_1, \ldots, \tilde{p}_m) \) such that, for all \( i \), \( \tilde{p}_i \)
is the unique value of \( p_i \) that maximises \( \pi_i(\tilde{p}_1, \ldots, \tilde{p}_{i-1}, p_i, \tilde{p}_{i+1}, \ldots, \tilde{p}_m, w_i) \).\(^11\)
The associated vector of equilibrium output levels is denoted by \( \tilde{q}(w) = (\tilde{q}_1, \ldots, \tilde{q}_m) \). We assume that \( \tilde{p}_i(w) \) and \( \tilde{q}_i(w) \) are both twice continuously
differentiable.

The upstream supplier’s profit is denoted by \( \pi_u(w) \) where:

\[
\pi_u(w) = \sum_{i=1}^{m} (w_i \tilde{q}_i(w)) - C \left( \sum_{i=1}^{m} \tilde{q}_i(w) \right)
\]

and \( C(\cdot) \) is the twice continuously differentiable cost function faced by the
input supplier. We assume that \( C' \) and \( C'' \) are both non-negative.

By our assumptions \( \pi_u(w) \) is twice continuously differentiable. To ensure
that a unique profit maximizing input price vector always exists, we assume
that \( \pi_u(w) \) is strictly concave.

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\(^9\)This normalization means that, even though the products sold by each downstream
firm can be different, the sum of these outputs is meaningful in terms of units of input.

\(^10\)Of course, production by firm \( i \) may also involve other inputs, however we do not need
to specify the overall production technology for downstream firm \( i \).

\(^11\)We only consider input prices where all firms produce positive levels of output in
equilibrium. This is to avoid trivial situations where a reduction in the input price to one
downstream firm could result in a rise in the input price to another downstream firm (a
‘waterbed effect’) where the firm facing the higher price does not produce any output.
The upstream supplier will set $w$ to maximize profits $\pi_u$. The upstream firm may, however, face a set of $n$ linear constraints when setting prices, of the form $w_j \leq \bar{w}_j$ for downstream firm $j \in \{1, \ldots, n\}$. These constraints capture any countervailing power. For example, a particular firm $j \in \{1, \ldots, n\}$ might have an outside option that would enable it to procure the input from an alternative supplier at a price $\bar{w}_j$. This option could be unique to firm $j$ and might reflect that it operates in multiple markets and can bring in the relevant input from an outside market. Alternatively, because of its size, firm $j$ might have an option to develop its own internal supply of the input and the minimum market price that makes such supply viable is $\bar{w}_j$.

Timing is as follows:

$t = 1$ The upstream supplier sets the input price vector $w$.

$t = 2$ Each downstream firm $i$ observes all input prices, makes its input purchases, and independently sets its price $p_i$ given its expectations of the behaviour of all other downstream firms.

We can summarize the outcome of the second stage of this game by the price and output vectors $\hat{p}(w)$ and $\hat{q}(w)$. At the first stage of the game the upstream supplier will set $w$ to maximize $\pi_u(w)$ subject to $w_j \leq \bar{w}_j$ for $j = 1, \ldots, n$. We denote the (unique) outcome of this game by $w^0$, $\hat{p}^0$ and $\hat{q}^0$ with upstream profit denoted by $\pi_u^0$.

### 3 The waterbed effect

To analyze the waterbed effect, consider firm 1 and assume that the input price constraint $w_1 \leq \bar{w}_1$ binds at $w_1^0$. In other words, the constraint on $w_1$ is $w_1 \leq w_1^0$.\footnote{Note that this is without loss of generality as, if the constraint does not bind initially (i.e. $w_1^0 < \bar{w}_1$) then we can introduce a new constraint $w_1 \leq w_1^0$. This constraint does not}$
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\[ i = 2, \ldots, m \] when the constraint on \( w_1 \) tightens. To do this, we consider the sign of \( \partial w_i / \partial w_1 \) for \( i = 2, \ldots, n \). For the waterbed effect to arise — so that the input price for one downstream firm rises when the input price paid by another firm falls — this derivative must be negative for some \( i \).

Let \( \lambda_j \) be the Lagrange multiplier associated with the input price constraint on firm \( j \). The initial vector of input prices \( w^0 \) will solve the \( m + n \) first order conditions:

\[
\tilde{q}_j + \sum_{i=1}^{m} \left( w_i \frac{\partial \tilde{q}_i}{\partial w_j} \right) - C'(\tilde{Q}) \left( \sum_{i=1}^{m} \frac{\partial \tilde{q}_i}{\partial w_j} \right) - \lambda_j = 0 \quad j = 1, \ldots, n
\]

\[
\tilde{q}_k + \sum_{i=1}^{m} \left( w_i \frac{\partial \tilde{q}_i}{\partial w_k} \right) - C'(\tilde{Q}) \left( \sum_{i=1}^{m} \frac{\partial \tilde{q}_i}{\partial w_k} \right) = 0 \quad k = n + 1, \ldots, m
\]

\[
\lambda_j (w_j - \bar{w}_j) = 0 \quad \lambda_j \geq 0 \quad j = 1, \ldots, n
\]

where \( \tilde{Q} = \sum_{i=1}^{m} \tilde{q}_i \).

As these first order conditions hold for all \( \bar{w}_1 \), we can use standard methods of comparative statics to solve for \( \partial w_i / \partial \bar{w}_1 \) for all \( i = 2, \ldots, n \).

At the initial outcome, \( (w^0, \tilde{p}^0, \tilde{q}^0) \), some firms \( j \in \{2, \ldots, n\} \), will have binding input price constraints with \( w_j = \bar{w}_j \) and \( \lambda_j > 0 \). For other firms \( j \in \{2, \ldots, n\} \) the input price constraint may be slack with \( \lambda_j = 0 \). Note that if \( \lambda_j = 0 \) for a specific firm \( j \), then the first order conditions for that firm are the same as the first order condition for a firm \( k \in \{n + 1, \ldots, m\} \).

As such, without loss of generality, we can reorder firms so that for all firms \( j = 1, \ldots, \bar{n}, \lambda_j > 0 \) at the initial outcome, while firms \( k = \bar{n} + 1, \ldots, m \) can be treated as not having a (binding) input price constraint at the initial

alter the initial profit maximizing choice of input prices for the upstream firm but means that we can now consider the situation where the constraint on \( w_1 \) tightens and alters the upstream firm’s profit maximizing choice of input prices.
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outcome, where \( \tilde{n} \leq n \).

Totally differentiating the first order conditions we get:

\[
\sum_{l=1}^{m} \left[ \frac{\partial \tilde{q}_{ij}}{\partial w_l} + \frac{\partial \tilde{q}_{il}}{\partial w_j} + \sum_{i=1}^{m} \left( w_i \frac{\partial^2 \tilde{q}_{ij}}{\partial w_l \partial w_j} \right) - \frac{C''(\tilde{Q})}{\partial \tilde{Q}} \frac{\partial \tilde{Q}}{\partial w_l} \right] dw_l = 0 \quad j = 1, \ldots, \tilde{n}
\]

\[
\sum_{l=1}^{m} \left[ \frac{\partial \tilde{q}_{ik}}{\partial w_l} + \frac{\partial \tilde{q}_{il}}{\partial w_k} + \sum_{i=1}^{m} \left( w_i \frac{\partial^2 \tilde{q}_{ik}}{\partial w_l \partial w_k} \right) - \frac{C''(\tilde{Q})}{\partial \tilde{Q}} \frac{\partial \tilde{Q}}{\partial w_k} \right] dw_l = 0 \quad k = \tilde{n} + 1, \ldots, m
\]

\[
dw_1 = d\bar{w}_1 \quad \text{and} \quad dw_j = 0 \quad j = 2, \ldots, \tilde{n}
\]

This system of \((m + \tilde{n})\) simultaneous equations can be solved to determine the sign of the \((m - \tilde{n})\) partial derivatives, \((\partial w_k / \partial w_1)\). If one or more of these derivatives is negative, so that \(w_k\) increases as \(w_1\) decreases, then this represents a ‘waterbed effect’ where the input supplier will find it profit maximizing to raise the input price to one downstream firm as a result of another downstream firm requiring a lower input price.

Solving the complete system is complex. However, significant intuition about if and when the waterbed effect will arise can be gained by focussing on situations with small numbers of downstream competitors. In section 4 we look at the situation of downstream duopoly and derive the condition for the waterbed effect to arise in this situation. Then, in section 5 we extend the analysis to three downstream competitors. We show that when only one of the downstream competitors has a constrained input price, the conditions

\[^{13}\text{Note that there could be a non-generic case where } \lambda_j = 0 \text{ and } w_j = \bar{w}_j \text{ at the initial outcome because } \bar{w}_j \text{ just happens to coincide with the profit maximizing input price for firm } j. \text{ We ignore such non-generic cases in what follows.}\]
for the waterbed effect are simply a natural extension of the duopoly case. In section 5 we also consider the situation where two of the downstream competitors have constrained input prices.

4 The case of two downstream firms

Much of the economic insight into the waterbed effect can be derived from the case of two downstream competitors \((m = 2)\). In this situation, \(\tilde{n} = 1\). The input supplier faces a binding constraint on the input price it can charge downstream firm \(i = 1\) but can freely set the input price for downstream firm \(i = 2\).

When \(m = 2\) the system of first order conditions reduces to a single relevant equation together with the constraint that \(dw_1 = d\bar{w}_1\).\(^{14}\) Solving this equation, and noting the second order conditions, the sign of \((\partial w_2/\partial w_1)\) is the same as the sign of:

\[
\frac{\partial \tilde{q}_2}{\partial w_1} + \frac{\partial \tilde{q}_1}{\partial w_2} - C''(\tilde{Q}) \left( \frac{\partial \tilde{Q}}{\partial w_2} \right) \left( \frac{\partial \tilde{Q}}{\partial w_1} \right) + \sum_{i=1}^{2} \left( \left( w_i - C'(\tilde{Q}) \right) \frac{\partial^2 \tilde{q}_i}{\partial w_2 \partial w_1} \right)
\]

A waterbed effect (i.e. \((\partial w_2/\partial w_1) < 0\)) will only arise if (1) is negative.

We can break (1) into three parts: a competition effect, a cost effect and an elasticity effect.

**The Competition Effect:** Consider the first part of (1), \(\frac{\partial \tilde{q}_2}{\partial w_1} + \frac{\partial \tilde{q}_1}{\partial w_2}\). This represents the change in sales of the input to one downstream firm when the input price of the other downstream firm changes. Thus \((\partial \tilde{q}_2/\partial w_1)\) considers how the equilibrium sales of the second downstream firm change when the input price paid by the first downstream firm changes.

We would expect \((\partial \tilde{q}_2/\partial w_1)\) to be positive if the two downstream firms are competitors, negative if the two downstream firms sell complementary products and zero if the two downstream firms sell independent products.

\(^{14}\)There is of course a further equation that determines \(d\lambda_1\).
Thus, a rise in $w_1$ will tend to raise the equilibrium price $\tilde{p}_1$ and lower equilibrium sales $\tilde{q}_1$ for firm 1. If firm 2 is a competitor to firm 1 then we would expect these changes to increase firm 2’s equilibrium sales $\tilde{q}_2$. We would expect the reverse to occur if firm 2’s product is complementary with firm 1. The same sign will apply to the other derivative ($\partial q_1/\partial w_2$).

To understand these terms, suppose the two downstream firms compete. This is the situation most commonly considered for a ‘waterbed’ effect. Then $(\partial q_2/\partial w_1)$ is positive, which acts against a waterbed effect. When the input price to downstream firm 1 falls, this reduces the ability of the downstream rival, firm 2, to compete. Given its lower input price, firm 1 will tend to reduce the price of its downstream product and to increase its level of sales. From the perspective of the input supplier, the (derived) demand for its product from firm 2 will fall, reducing the profit-maximizing price that it can charge firm 2. Thus, the first two terms capture a feedback from a reduction in the input price paid by one firm through to the profit maximizing input price charged to its competitive rival.

The competition effect acts against the waterbed effect for competitive downstream firms. Rather than a fall in the input price to one downstream firm raising the input price to competitors, the input price to competitors will also tend to fall. These terms suggest that, to the degree a waterbed effect exists, it may be relevant for downstream firms that produce complementary outputs using the same input, not for downstream competitors.

**The Cost Effect:** The second part of (1), $-C''(\tilde{Q}) \left( \frac{\partial \tilde{Q}}{\partial w_2} \right) \left( \frac{\partial \tilde{Q}}{\partial w_1} \right)$, represents the effect on the input supplier’s production costs when output changes as a consequence of the fall in the input price it can charge downstream firm 1. We would expect $(\partial \tilde{Q}/\partial w_1)$ and $(\partial \tilde{Q}/\partial w_2)$ to both be the same sign so that, multiplied together, they are positive.\(^{15}\) By assumption, $C''(\tilde{Q}) \geq 0$.

\(^{15}\)Indeed, we would expect each of $(\partial \tilde{Q}/\partial w_1)$ and $(\partial \tilde{Q}/\partial w_1)$ to be negative so that a reduction in an input price raises total equilibrium output.
Thus, we expect the total cost effect, as given by 

\[-C''(\tilde{Q}) \left( \frac{\partial \tilde{Q}}{\partial w_2} \right) \left( \frac{\partial \tilde{Q}}{\partial w_1} \right),\]

to be negative.

The cost effect captures the ‘common intuition’ of the water bed effect. As one downstream firm is able to reduce the price that it pays for the input, this will ‘force’ the supplier to raise the input price to other downstream firms in order to ‘cover its costs’. However, the effect is more subtle than this ‘common intuition’. It reflects that a reduction in price to one downstream firm will increase input sales to that firm, which raises the marginal production cost to supplying all downstream firms \textit{ceteris paribus}. Thus the cost effect operates because a fall in the input price to one firm raises total sales of the input. Further, it operates because these increased sales involve an increase in marginal cost. Interestingly, the cost effect operates regardless of the nature of downstream competition. Thus the cost effect can support a ‘waterbed effect’ regardless of whether firms compete downstream or not.

**The Elasticity Effect:** The final part of (1), \( \sum_{i=1}^{2} \left( \left( w_i - C'(\tilde{Q}) \right) \frac{\partial^2 \tilde{q}_i}{\partial w_2 \partial w_1} \right), \) captures the effect on the profit maximizing input prices due to a change in the slope of the (derived) demand curves facing the input supplier. The terms \( (w_i - C'(\tilde{Q})), i = 1, 2 \) represent the per unit profit the supplier receives from selling a unit of input to either downstream firm. The cross derivative terms \( (\partial^2 \tilde{q}_i/\partial w_2 \partial w_1), i = 1, 2 \), capture the effect of the change in input price for one downstream firm on the sensitivity of demand for the other downstream firm. For example, if a fall in \( w_1 \) makes the (derived) input demand for downstream firm \( i = 2 \) less sensitive to a change in the input price, this provides an incentive for the input supplier to raise the input price \( w_2 \) when \( w_1 \) falls. In this situation, \( (\partial^2 \tilde{q}_2/\partial w_2 \partial w_1) < 0. \)

We would expect the input supplier to earn positive margins on the product it sells to downstream firms, so that \( (w_i - C'(\tilde{Q})) \) will be positive for \( i = 1, 2 \). However, \textit{apriori}, there is no reason to expect that the cross deriva-

\footnote{Remembering, of course, that \( (\partial \tilde{q}_2/\partial w_2) < 0. \)}
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tives \( \partial^2 \tilde{q}_i / \partial w_2 \partial w_1 \), \( i = 1, 2 \), will have a particular sign. Rather it will depend on the nature of downstream interaction.

Discussion of the two-firm case: The case of two downstream firms sheds significant light on the possibility of a waterbed. The analysis shows that the waterbed effect can occur and that it has solid economic underpinnings. Further, it highlights the relevant market factors that policy makers need to consider when debating both the likelihood and the consequences of the waterbed effect for particular industries.

For example, the two-firm case highlights that a waterbed effect is more likely when upstream supply costs are more convex, so if one firm achieves a lower input price this can lead to higher prices to other firms due to rising upstream marginal cost. In contrast, if upstream supply involves constant returns to scale technology, the marginal cost of supply does not change for the input supplier and the 'cost effect' is zero. Thus the likelihood of a waterbed effect depends directly on the nature of the upstream technology. However, this effect has nothing to do with the nature of downstream competition.

In fact, the two-firm case suggests that researchers may have been looking for the waterbed effect in the wrong place. Suppose for example, that the two downstream firms operate in separate markets. These may be firms that operate in different countries, or even firms that operate in geographically separate parts of the same country. In such a situation, there is no competition between the firms and we would expect both the competition and the elasticity effects to be zero (or close to zero). The effect on one downstream
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The firm’s input price as a result of a reduction in the other firm’s input price will depend on the ‘cost effect’. So long as the input supplier faces convex costs, there will be a waterbed effect, albeit that the size of this effect will depend on the exact technology.

While the two firm case sheds significant light on the waterbed effect, it is useful to consider the case of three downstream firms to see if anything changes.

5 The case of three downstream firms

In this section we consider the case of three downstream firms so that \( m = 3 \).

In subsection 5.1 we set \( \tilde{n} = 1 \). A fall in \( \bar{w}_1 \), lowering the input price for the first downstream firm, may lead to a profit maximizing increase in the input price for either of the other two downstream firms. A waterbed effect will arise if either \( \left( \partial w_2 / \partial w_1 \right) < 0 \) or \( \left( \partial w_3 / \partial w_1 \right) < 0 \).

In subsection 5.2 we consider the situation where \( \tilde{n} = 2 \) so that a drop in \( w_1 \) due to an exogenous tightening of the input price constraint for the first downstream firm can only lead to a change in the input price facing the third downstream firm. The second downstream firm’s input price constraint binds. In this situation, a waterbed effect will arise if \( \left( \partial w_3 / \partial w_1 \right) < 0 \).

Finally, in subsection 5.3 we consider the situation where \( \tilde{n} = 2 \) but there are linkages between input price constraints, so that a change in \( \bar{w}_1 \) can lead to a change in \( \bar{w}_2 \). Again, we consider the implications for a waterbed effect.

5.1 The waterbed effect when there are two unconstrained downstream firms

Consider the case where \( m = 3 \) and, at the initial equilibrium \( w^0, \bar{p}^0 \) and \( \bar{q}^0 \), \( \tilde{n} = 1 \). We wish to consider the effect of a change in the binding input price constraint for the first downstream firm, \( dw_1 = d\bar{w}_1 \). Our interest is in the
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The sign of the change of the profit maximizing input price for the second and third downstream firms; \( (\partial w_2/\partial w_1) \) and \( (\partial w_3/\partial w_1) \).

From the general system of first order equations, we obtain:

\[
\begin{align*}
\left[ \frac{\partial q_2}{\partial w_1} + \frac{\partial q_1}{\partial w_2} + \sum_{i=1}^3 \left( w_i \frac{\partial^2 q_i}{\partial w_2 \partial w_1} \right) - C''(\tilde{Q}) \left( \frac{\partial Q}{\partial w_2} \right) \left( \frac{\partial q_i}{\partial w_1} - C'(\tilde{Q}) \left( \frac{\partial^2 q_i}{\partial w_2 \partial w_1} \right) \right) \right] dw_1 \\
+ \left[ 2 \frac{\partial q_2}{\partial w_2} + \sum_{i=1}^3 \left( w_i \frac{\partial^2 q_i}{\partial w_2 \partial w_1} \right) - C''(\tilde{Q}) \left( \frac{\partial Q}{\partial w_2} \right) \left( \frac{\partial^2 q_i}{\partial w_2 \partial w_1} \right) \right] dw_2 \\
+ \left[ \frac{\partial q_3}{\partial w_2} + \sum_{i=1}^3 \left( w_i \frac{\partial^2 q_i}{\partial w_2 \partial w_3} \right) - C''(\tilde{Q}) \left( \frac{\partial Q}{\partial w_2} \right) \left( \frac{\partial^2 q_i}{\partial w_2 \partial w_3} \right) \right] dw_3 = 0
\end{align*}
\]

Using Cramer’s rule to solve this system of equations, the sign of \( (\partial w_2/\partial w_1) \) is given by the sign of:

\[
- \left[ \frac{\partial q_2}{\partial w_1} + \frac{\partial q_1}{\partial w_2} - C''(\tilde{Q}) \left( \frac{\partial Q}{\partial w_2} \right) \left( \frac{\partial q_i}{\partial w_1} + \sum_{i=1}^3 \left( \left( w_i - C'(\tilde{Q}) \right) \frac{\partial^2 q_i}{\partial w_2 \partial w_1} \right) \right) \right] \times
\left[ 2 \frac{\partial q_2}{\partial w_2} - C''(\tilde{Q}) \left( \frac{\partial Q}{\partial w_2} \right) \left( \frac{\partial q_i}{\partial w_1} + \sum_{i=1}^3 \left( \left( w_i - C'(\tilde{Q}) \right) \frac{\partial^2 q_i}{\partial w_2 \partial w_1} \right) \right) \right]
\]

\[
+ \left[ \frac{\partial q_2}{\partial w_2} + \frac{\partial q_1}{\partial w_2} - C''(\tilde{Q}) \left( \frac{\partial Q}{\partial w_2} \right) \left( \frac{\partial q_i}{\partial w_1} \right) \times \sum_{i=1}^3 \left( \left( w_i - C'(\tilde{Q}) \right) \frac{\partial^2 q_i}{\partial w_2 \partial w_3} \right) \right]
\]

Note that by the second order conditions that

\[
\left[ 2 \frac{\partial q_2}{\partial w_3} - C''(\tilde{Q}) \left( \frac{\partial Q}{\partial w_3} \right) \left( \frac{\partial q_i}{\partial w_1} + \sum_{i=1}^3 \left( \left( w_i - C'(\tilde{Q}) \right) \frac{\partial^2 q_i}{\partial w_2 \partial w_1} \right) \right) \right] < 0 \quad (2)
\]

We can break the sign condition for \( (\partial w_2/\partial w_1) \) into two parts. Given (2) the sign of \( (\partial w_2/\partial w_1) \) will tend to be the same as the sign of

\[
\frac{\partial q_2}{\partial w_1} + \frac{\partial q_1}{\partial w_2} - C''(\tilde{Q}) \left( \frac{\partial Q}{\partial w_2} \right) \left( \frac{\partial q_i}{\partial w_1} \right) + \sum_{i=1}^3 \left( \left( w_i - C'(\tilde{Q}) \right) \frac{\partial^2 q_i}{\partial w_2 \partial w_1} \right)
\]
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But note that this is almost identical to (1) from the two-firm case. The only difference is that there is an extra term in the ‘elasticity effect’ relating to the sensitivity of (derived) demand for the input by the third downstream firm. In other words, the logic of the two firm case continues to hold for the first part of the condition with three downstream firms.

Second, the sign of \( \frac{\partial w_2}{\partial w_1} \) will tend to be the same as the sign of:

\[
\left[ \frac{\partial \tilde{q}_3}{\partial w_2} + \frac{\partial \tilde{q}_2}{\partial w_3} - C''(\tilde{Q}) \left( \frac{\partial \tilde{Q}}{\partial w_3} \right) \left( \frac{\partial \tilde{Q}}{\partial w_2} \right) + \sum_{i=1}^{3} \left( (w_i - C'(\tilde{Q})) \frac{\partial^2 \tilde{q}_i}{\partial w_i \partial w_2} \right) \right] \times \left[ \frac{\partial \tilde{q}_3}{\partial w_1} + \frac{\partial \tilde{q}_1}{\partial w_3} - C''(\tilde{Q}) \left( \frac{\partial \tilde{Q}}{\partial w_3} \right) \left( \frac{\partial \tilde{Q}}{\partial w_1} \right) + \sum_{i=1}^{3} \left( (w_i - C'(\tilde{Q})) \frac{\partial^2 \tilde{q}_i}{\partial w_i \partial w_1} \right) \right]
\]

We can interpret the first bracketed term as the effect on \( w_2 \) of a change in the input price to firm 3 while the second bracketed term is the direct effect of a change in \( w_1 \) on \( w_3 \). Thus, together, this term can be interpreted as the indirect effect of a change in \( w_1 \) on the input price of firm 2, through the profit maximizing change in \( w_3 \).

An equivalent expression can be derived to sign \( \frac{\partial w_3}{\partial w_1} \)

The economic intuition from the two-firm case is validated in the three firm case. As before, if downstream firms are competitors, all derivatives \( (\partial \tilde{q}_i / \partial w_j) \) will be positive, which acts against the waterbed effect. Similarly, if upstream production technology is constant returns to scale, this reduces the likelihood of a waterbed effect.

The indirect effect, however, shows that any waterbed effect may be moderated with two ‘unconstrained’ downstream firms. To see this suppose that all ‘direct’ effects are negative in the sense that:

\[
\frac{\partial \tilde{q}_k}{\partial w_j} + \frac{\partial \tilde{q}_j}{\partial w_k} - C''(\tilde{Q}) \left( \frac{\partial \tilde{Q}}{\partial w_k} \right) \left( \frac{\partial \tilde{Q}}{\partial w_j} \right) + \sum_{i=1}^{3} \left( (w_i - C'(\tilde{Q})) \frac{\partial^2 \tilde{q}_i}{\partial w_k \partial w_j} \right) < 0
\]

for all \( j, k = 1, 2 \) or 3 with \( j \neq k \). The indirect effect will then involve the multiple of two negative terms which tends to moderate the ‘direct’ waterbed effect.
If the ‘direct’ effects are all negative, so a reduction in \( w_1 \) tends to raise the profit maximizing input price for downstream firm \( i = 2 \) and \( i = 3 \), then as \( w_3 \) rises the waterbed effect starts to operate in reverse on \( w_2 \). The rise in \( w_3 \) tends to moderate the size of any rise in \( w_2 \) and vice versa. Of course, our analysis here cannot determine the absolute size of \( (\partial w_2/\partial w_1) \) or \( (\partial w_3/\partial w_1) \) - that would require specific numerical analysis.

In summary, when there are multiple unconstrained downstream firms, the logic of the waterbed effect for the two-firm case continues to hold. However, there are offsetting indirect effects that will interact with the direct effects. In particular, if all downstream firms face a ‘direct’ waterbed effect, then the indirect effects will act to offset the direct effects.

5.2 The waterbed effect when there is one unconstrained downstream firm

Consider the case where \( m = 3 \) and, at the initial equilibrium \( w^0, \hat{p}^0 \) and \( \hat{q}^0, \tilde{n} = 2 \). We wish to consider the effect of a change in the binding input price constraint on the first downstream firm \( dw_1 = d\hat{w}_1 \) holding the input price constraint on the second downstream firm unchanged; \( dw_2 = 0 \). Our interest is in the sign of the change of the profit maximizing input price for the third downstream firm; \( (\partial w_3/\partial w_1) \).

It is easy to see that in this situation the sign of \( (\partial w_3/\partial w_1) \) is the same as the sign of:

\[
\frac{\partial \tilde{q}_3}{\partial w_1} + \frac{\partial \tilde{q}_1}{\partial w_3} - C''(\hat{Q}) \left( \frac{\partial \hat{Q}}{\partial w_3} \right) \left( \frac{\partial \hat{Q}}{\partial w_1} \right) + \sum_{i=1}^{3} \left( \left( w_i - C'(\hat{Q}) \right) \frac{\partial^2 \tilde{q}_i}{\partial w_3 \partial w_1} \right) \tag{3}
\]

This is equivalent to (1) except for an extra ‘indirect’ effect relating to the second constrained downstream firm, \( i = 2 \), that is included in the ‘elasticity term’. Thus the logic behind the waterbed effect in this case is similar to the two-firm case.
5.3 The waterbed effect when input price constraints are linked

Finally, it is possible that there may be links between the input price constraints facing the upstream supplier. For example, a tightening of the input price constraint relating to downstream firm \( i = 1 \) may also alter an input constraint relating to another downstream firm.

Input price constraints could be linked for a variety of reasons. For example, two downstream firms may be able to (legally) enter an agreement that provides them with the option of jointly establishing an alternative source of supply that would be uneconomic for either downstream firm by themselves. In this situation, the price constraint faced by the upstream supplier when setting input prices for either of these downstream firms will tighten due to the agreement.

Alternatively, constraints may move in opposite directions. For example, if one downstream firm merges with a vertically integrated firm in a different country, this may enhance its own option of gaining alternative input supply, but may make the threat of input supply to other downstream firms from the vertically integrated company less credible.

To analyze this situation, consider the case where \( m = 3 \) and at the initial equilibrium \( w^0, \tilde{p}^0 \) and \( \tilde{q}^0, \tilde{n} = 2 \). We wish to consider the effect of a change in the binding input price constraint on the first downstream firm \( dw_1 = d\tilde{w}_1 \) that is connected to the change in the input price constraint on the second downstream firm. Thus, \( dw_2 = \delta d\tilde{w}_1 \) where \( \delta \) can be either positive or negative. Clearly the case with \( \delta = 0 \) is identical to subsection 5.2 above. Again, our interest is in the sign of the change of the profit maximizing input price for the third downstream firm; \( (\partial w_3/\partial w_1) \).

Solving the system of first-order equations in this case we see that the
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The sign of \((\partial w_3 / \partial w_1)\) is the same as the sign of:

\[
\left[ \frac{\partial \tilde{q}}{\partial w_1} + \frac{\partial \tilde{q}}{\partial w_3} - C''(\tilde{Q}) \left( \frac{\partial \tilde{q}}{\partial w_3} \right) \left( \frac{\partial \tilde{q}}{\partial w_1} \right) + \sum_{i=1}^{3} \left( \left( w_i - C'(\tilde{Q}) \right) \frac{\partial^2 \tilde{q}}{\partial w_3 \partial w_1} \right) \right]
\]

\[\text{(4)}\]

The first part of (4) is equivalent to (3). It gives the direct effect on \(w_3\) due to the change in \(w_1\). The second part of (4) represents the effect on \(w_3\) due to a change in \(w_2\). It is weighted by \(\delta\) to reflect the link between the change in \(w_1\) and the change in \(w_2\). Depending on the sign of \(\delta\), the linking of the two price constraints can exacerbate or reduce any waterbed effect that arises when \(w_1\) falls.

Finally, there is of course a waterbed effect on downstream firm 2 whenever \(\delta\) is negative. In this situation, a fall in \(w_1\) will change the constraint on the second downstream firm allowing the input supplier to raise the price to this downstream firm.

In our model any linking of constraints is exogenous. However, as discussed in section 1, this linkage plays a key role in the analysis of the waterbed effect, for example, by Inderst and Valletti (2011).

While a price change that arises due to linkage between pricing constraints may be of interest in its own right, in our opinion, the key argument about the waterbed effect has arisen where such linkage is absent. In particular the debate has focussed on small firms (such as small retailers) who have little if any outside option for an input, and how their input prices will alter when a larger firm is able to exert buyer power on the relevant input supplier. As such, our model has focused on these situations and highlighted if, and when, such small firms will face a rise in their input prices.

6 Conclusion

This paper has presented a simple but parsimonious model to analyse the relationship between countervailing power and downstream pricing. We have
focussed on situations where one downstream firm has increased counter-vailing power that enables it to gain a lower input price from an upstream supplier. Our focus has been on the effect of such a change on the profit maximizing prices that the supplier sets for other downstream firms. In particular, we have considered whether or not a waterbed effect could arise: where a reduction in one downstream firm’s input price due to increased countervailing power results in a higher input price to one or more of the other downstream firms.

Our model has shown that the presence or absence of any waterbed effect will depend on the interplay of three interrelated changes. First, a waterbed effect is less likely to arise when a downstream firm is a competitor to the firm that has increased its countervailing power. This ‘competition effect’ arises because a lower input price for one downstream firm moderates the input demand of its competitors. In contrast, if downstream firms use the same input to produce complementary products, then a waterbed effect is more likely.

Second, a waterbed effect is more likely when upstream production costs are convex. Increased countervailing power for one downstream firm means that this firm receives a lower input price and purchases more of the input. But with convex production costs, the increase in sales of the input tends to raise the marginal cost for the input supplier. This, in turn, raises the profit maximizing price the supplier will set for all other downstream firms.

Importantly, this ‘cost effect’ does not depend on downstream competition or complementarity. It arises whenever the downstream firms buy the same input and production costs for the input are convex. Thus an ‘independent’ downstream firm, in a different location or market to the downstream firm with increased countervailing power, could face a ‘waterbed effect’ due to the convexity of upstream costs.

Third, a change in the input price paid by one downstream firm can lead to changes in the elasticity of input demand for other downstream firms.
These changes can lead to a rise or fall in the profit maximizing price that the supplier sets for the relevant downstream firms. It could lead to a ‘waterbed effect’, or the reverse. Without knowing the details of downstream demand, the influence of this ‘elasticity effect’ is ambiguous.

This paper is complementary to other papers that have considered the waterbed effect. This other work has focussed on price changes that originate from ‘links’ between bargaining outcomes. In other words, these papers have considered situations where multiple downstream firms have countervailing power and a change in the countervailing power for one downstream firm has implications for the countervailing power of other downstream firms. Our model is able to include such links, so that an increase in countervailing power for one downstream firm can directly effect the price constraint that the supplier faces for another downstream firm. However, in our model such linkages are ‘black boxed’. Other work has considered how and why these linkages can arise.

In contrast, this paper focuses on input price changes for firms that have no countervailing power. In this sense, our model captures some of the concerns expressed in the public debate on the waterbed effect, where the fear is that countervailing power exercised by one or more ‘large’ downstream firms will lead to higher input prices for small downstream firms that have no countervailing power.17

The model presented in this paper provides useful guidance for policy makers concerned about the ‘waterbed effect’. For example, if downstream firms are strong competitors, upstream production technology is constant returns to scale and final consumers are ‘similar’, so that significant changes in the elasticity of the derived input demand are unlikely, then a ‘waterbed effect’ is unlikely.18

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17 Of course, the model presented in this paper does not require size to be associated with countervailing power.

18 The caveat here is that a waterbed effect may arise through linkages in bargaining
In contrast, a waterbed effect is more likely when upstream production is characterized by capacity constraints or highly convex costs. But this effect on input prices reflects the need for the input supplier to ration output among downstream firms. It is independent of the nature of downstream interaction.

Indeed, the model presented in this paper suggests that policy makers may have been looking for the waterbed effect in the wrong place. Downstream competition militates against the waterbed effect. Thus, a waterbed effect may be more likely where downstream firms operate in different markets or sell complementary products.

References


constraints. As noted, the model presented here complements the existing literature that considers these linkages.
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