On the Time Inconsistency of Optimal Monetary and Fiscal Policies With Many Consumer Goods

Begoña Dominguez* and Pedro Gomís-Porqueras†

Abstract
This paper studies optimal monetary and fiscal policies in an economy à la Lucas and Stokey (1983) and Lagos and Wright (2005) with multiple cash and credit goods. We show that optimal policies are in general time inconsistent due to insufficient number of instruments to influence future government decisions. There are two important cases where time consistency can be restored. First, if taxes in the decentralized anonymous markets are not available, the multipliers on the decentralization constraints can be utilized as additional instruments to ensure time consistency. Second, if taxes in decentralized markets are available, time consistency arises when the different cash goods satisfy the conditions necessary for optimal uniform taxation.

JEL Codes: C70, E40, E61, E62, H21.
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*Corresponding author: Begoña Dominguez. School of Economics, The University of Queensland, Colin Clark Building(39), St Lucia, Brisbane, Qld 4072, Australia. E-mail: b.dominguez@uq.edu.au

†Department of Economics, Monash University, Caulfield Campus, Room H4.31, Caulfield East, Vic 3145, Australia. E-mail: pedro.gomis@monash.edu

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1 Introduction

Can governments induce commitment to optimal monetary and fiscal policies? In an environment where fiat money is essential, this paper examines this question and demonstrates that when there are multiple government policy decisions per period affecting nominal variables, optimal monetary and fiscal policies are in general time inconsistent.

This paper studies optimal fiscal and monetary policies in an environment that blends the shopper-worker structure of Lucas and Stokey (1983) with the sequential nature of trade, lack of enforcement and informational frictions of Lagos and Wright (2005). In every period agents trade in two different markets. First they trade in a decentralized anonymous market (DM) where the only incentive compatible form of payment is fiat money.\(^1\) Then agents trade in a centralized market (CM) where any form of payment is possible. Agents consume positive amounts of several consumer goods in both markets. These DM and CM goods are the same in terms of their product characteristics, but they are intrinsically different as they are traded in different locations and require different forms of payment.

In order to finance an exogenous government spending, the benevolent government chooses a general tax rate on all CM goods, a specific tax rate on each CM good (other than the benchmark CM good 1), nominal interest rates that affect all DM goods, a specific tax rate on each DM good (other than the benchmark DM good 1), the initial level of prices, and the issues of nominal and real government bonds. Real bonds can be indexed to each CM good. However, as in Lucas and Stokey (1983), bonds indexed to DM goods are not allowed as that would not be consistent with having DM goods bought at decentralized markets where agents are anonymous.\(^3\) This is a crucial assumption for the results of the paper as well as for the existence of money.

Our main findings are as follows. We show that optimal policies are in general time inconsistent because governments do not have a sufficient number of instruments to influence all future government decisions. There are two important cases where time consistency can be restored.

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\(^1\) Kocherlakota (1998) laid out the critical frictions for fiat money to be essential. He showed that if any of the following elements in the economic environment are absent: (i) recordkeeping over individual trading histories (“memory”), (ii) public communication of histories and (iii) sufficient enforcement (or punishment), then credit between buyer and seller is not incentive compatible so money is essential. The search literature uses the term “anonymity” to encompass these three frictions.

\(^2\) We rule out bonds as a medium of exchange. For that to hold in equilibrium, the literature argues that bonds must have some illiquidity relative to fiat money. Wallace (1983) and Aiyagari, Wallace and Wright (1996) argue that bonds are illiquid because of legal restrictions that prohibit the issuing of bonds of small denomination. On the other hand, Li, Rocheteau and Weill (2011) emphasize informational asymmetries between assets that affect recognizability properties of fiat money and bonds determining the liquidity of assets as an equilibrium outcome endogenously.

\(^3\) In an economy with one cash good and one credit good, Lucas and Stokey (1983) write: “If such securities were available, however, they could be used by agents to circumvent the use of currency altogether, converting the system directly into the two-good barter economy ... This would conflict with our interpretation of cash goods as being anonymously purchased in spot markets only. To maintain the monetary interpretation of the model, then, direct claims to cash goods in “real” terms will be ruled out.”
First, if taxes in the decentralized anonymous markets are not available, the multipliers on the
decentralization constraints can be used as additional instruments to ensure time consistency.
Second, if taxes in decentralized markets are available, time consistency arises when the differ-
ent decentralized market goods satisfy the conditions necessary for optimal uniform taxation.
Therefore, the frictions that make money essential, anonymity and lack of enforcement, which
are important for the feasibility of taxation in DM, are then also relevant for time consistency.
This is consistent with Wallace (2001) who points out that different approaches of modelling
money have important implications for other parts of the model. In our paper, informational
frictions that make fiat money essential also shape the debt and tax instruments available and
the sustainability of the resulting optimal fiscal and monetary policies.

This paper builds on the existing literature on the time consistency of optimal monetary and
fiscal policies.\(^4\) Since the seminal work of Calvo (1978), the severity of the time inconsistency
problem of monetary and fiscal policies has been widely recognized and studied. Lucas and
Stokey (1983) show that the incentives to inflate away the initial stock of nominal bonds and
money are so severe that optimal monetary and fiscal policies cannot be made time consistent.
Recently, two papers have proposed solutions to this classic problem in public finance. Alvarez,
Kehoe and Neumeyer (2004) find that optimal monetary and fiscal policies are time consistent
when the Friedman rule is optimal, as their monetary economy becomes isomorphic to a real
economy. Persson, Persson and Svensson (2006) have demonstrated that time consistency in
optimal monetary and fiscal policies can be achieved independently of the optimality of the
Friedman rule whenever surprised inflation imposes some direct costs to agents.

This paper relates to both articles. As in Persson et al. (2006), we assume that consumption
in decentralized markets takes place before centralized market trades. This timing assumption
implies that surprise inflation has a direct cost in the economy and the optimal initial level of
prices is finite. However, relative to Persson et al. (2006), our environment has multiple goods
that require fiat money to settle trade. Thus optimal policies cannot be made time consistent
with just one nominal bond. In contrast to Alvarez et al. (2004), in our environment the
Friedman rule is not always optimal. Nevertheless, even when the Friedman rule is optimal,
future fiscal choices in decentralized markets cannot be influenced by current governments. This
implies that optimal monetary and fiscal policies are time inconsistent even when the Friedman
rule is optimal.

This paper justifies and complements the work of Martín (2011) who considers a search
theoretic model of fiat money where agents face some limited commitment in some markets and
where the government can not commit to future policies. Martín (2011) studies time-consistent
Markov policies and finds that when net nominal government obligations are positive, there

\(^{4}\)Persson et al. (2006) provide a comprehensive review of the literature.
exists a unique steady state with positive net nominal government obligations, which is stable and time-consistent. For any initial level of debt, the welfare loss due to lack of commitment is small. Within the framework, Martín (2012a) studies the determination of government policy when aggregate shocks (expenditure and labor productivity) are present while considering various trading protocols in decentralized markets. The author finds that fiscal and monetary policies are distortionary, but long-run policy is not afflicted by time-consistency problems when different trading protocols are considered.\(^5\)

As in Martín (2011) and (2012a), this paper tries to improve our understanding of the inherent links between fiscal and monetary policies in the absence of commitment both by governments and by agents in some markets. By considering alternative approaches to modeling fiat money that emphasize different frictions in the economic environment, we are able to determine the robustness of the policy prescriptions prescribed by the previous literature.

The structure of the rest of the paper is as follows. Section 2 describes our model economy. Section 3 characterizes optimal fiscal and monetary policies with commitment. Section 4 studies the time consistency of the optimal policy. Section 5 concludes.

## 2 The environment

We consider an economy that incorporates elements of Lucas and Stokey (1983) and various ingredients of search-theoretic models of money. In particular, we consider the large household framework of Shi (1997) and the sequential nature of trade and informational frictions of Lagos and Wright (2005) making a medium of exchange essential.

Time is discrete and indexed by \(t\). The economy is comprised by a continuum of households that live forever and have a common discount factor \(\beta \in (0, 1)\) between time periods. Within each household, agents belong to a large family of measure one. Every period is comprised of two sub-periods or stages, day and night, which differ in terms of the enforcement and the informational frictions that agents face.

In the first stage (day), households trade in a decentralized market (DM). This market is characterized by anonymous trades, lack of enforcement, no record-keeping services and by a lack of double coincidence of wants. Given these assumptions, the only feasible trade in DM is the exchange of goods for fiat money.\(^6\) A fraction \(\kappa \in (0, 1)\) of household members are potential buyers (who can buy but not sell) and a fraction \((1 - \kappa)\) are potential sellers (who can sell but not buy). With probability \(\sigma \in (0, 1)\), agents (buyers and sellers) of all families are randomly shuffled.

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5Martín (2012b) considers a similar environment and evaluates the implied time-consistent Markov policies in explaining U.S. wartime. The author finds that during the period under analysis there were strong incentives to inflate as the government was faced by an increased stock of debt.

6Since there are no record keeping services, bonds cannot be used as a medium of exchange in DM.
assigned to different identical locations in DM. If agents are not assigned to any location, they cannot trade in DM for that period. Otherwise, buyers and sellers of each family are sent to different locations. Thus, as in Shi (1997), members of the same family cannot trade with each other. The matching technology is such that the measure of buyers and sellers is the same across all locations as in Lucas and Prescott (1974). In order to be closest to the cash and credit good economy of Lucas and Stokey (1983), we assume that the trading protocol in DM is competitive pricing. Once trades take place, all members of the household share the proceeds of the DM trade before moving into the next market.

In the second stage (night), households trade in a centralized market (CM). In this market, all members of the family can produce and consume CM goods as well as adjust their portfolio by rebalancing their money and (real and nominal) bond holdings. In this market there are no informational frictions nor lack of double coincidence of wants thus a medium of exchange is not essential.

2.1 Many Consumer Goods

In this environment we consider multiple consumer goods both in CM and DM. More specifically, individuals want to consume $I > 1$ distinct consumption goods in CM, denoted by the vector $X_t = (X_{1,t}, \ldots, X_{I,t})$, and $I > 1$ distinct consumption goods in DM, denoted by $x_t = (x_{1,t}, \ldots, x_{I,t})$. The $I$ goods in CM and DM are not perfectly substitutable within each sub-market and individuals require positive amounts of consumption of each good.

The key difference between CM and DM goods is the market in which they are purchased and therefore the available payments methods to settle trade. The first vector of CM goods, $X_t$, is purchased by any means of payment available in the economy. On the other hand, the vector of DM goods, $x_t$, can only be purchased with fiat money given the informational frictions in that market. We follow Lucas and Stokey (1983) in assuming that CM and DM goods cannot be singled out through observable “product” characteristics. However, the market, in which they are purchased, makes consumption $X_{i,t}$ different from consumption $x_{i,t}$. Moreover, these goods may have a different income elasticity of demand and may yield different utilities.\footnote{An example of what we have in mind is the following: a hot-dog sold at a stadium stand during a soccer match is a DM good and a hot-dog sold at the corner shop owned by a neighbour friend is a CM good. Both underlying consumption goods are identical, but they are sold at different markets with different payment methods. This makes both products in fact different and with a likely different demand elasticity.}

We allow governments to issue bonds indexed to the CM good $i$, for all $i = 1, \ldots, I$. However, as also argued in Lucas and Stokey (1983), we do not allow for bonds indexed to DM goods as that would conflict with having DM goods been bought at decentralized markets (where agents are anonymous and lack enforcement) and would make money non essential. As mentioned, this is a crucial assumption for the results of the paper as well as for the existence of money.
2.2 Preferences and Technologies

At any date \( t \), households have preferences over all of the \( I \) DM consumption goods \( (x_t = (x_{1,t}, \ldots, x_{I,t})) \), total effort exerted to produce all of the DM goods \( (l_t = \sum_{i=1}^{I} l_{i,t}) \), all of the \( I \) CM consumption goods \( (X_t = (X_{1,t}, \ldots, X_{I,t})) \) and total effort exerted to produce all of the CM goods \( (L_t = \sum_{i=1}^{I} L_{i,t}) \). These are given by:

\[
E_0 \sum_{t=0}^{\infty} \beta^t [u(x_t) - h(l_t) + U(X_t) - H(L_t)].
\] (1)

The instantaneous utility functions \( u(\cdot) \), \( U(\cdot) \), \( h(\cdot) \) and \( H(\cdot) \) are all strictly increasing and three times continuously differentiable. The functions \( u(\cdot) \) and \( U(\cdot) \) are strictly concave. For all \( i \), we assume that \( u_{x_i} = \infty \) whenever \( x_i = 0 \) and, equivalently, \( U_{X_i} = \infty \) whenever \( X_i = 0 \). Throughout the rest of the paper, subscript \( x_i \) denotes the derivative of the function with respect to \( x_i \). To simplify the analysis, both \( u(\cdot) \) and \( U(\cdot) \) are assumed to be additively separable, so that \( u_{x_i,x_j} = U_{X_i,X_j} = 0 \) for \( i \neq j \), with \( i, j \in 1, \ldots, I \).\(^8\) It is important to note that the functions \( u(\cdot) \) and \( U(\cdot) \) do not have to satisfy homotheticity, or have the same specific form. We further assume that the functions \( h(\cdot) \) and \( H(\cdot) \) are strictly convex and satisfy \( h(0) = H(0) = 0 \) and \( h_L(1) = H_L(1) = \infty \).\(^9\)

In DM all consumption goods are produced according to a constant returns technology \( f \), where labor is the only input and one unit of labor yields one unit of output, \( f(l_{i,t}) = l_{i,t} \). Recall that only a fraction of the family can actually produce these goods. On the other hand, in the CM, all members of the family can produce goods. Agents have also access to a linear production technology \( F \) that can transform labor into final goods so that \( F(L_{i,t}) = L_{i,t} \).

2.3 Solving the Agent’s Problem

Next we solve the agent’s problem recursively. Thus, we first solve the second stage (CM) problem taking the actions of the first stage as given. We then solve the first stage (DM) problem.

2.3.1 The Second Stage Problem

After trades take place in DM, agents enter the next submarket CM before the end of the period. Since in this market agents can both produce and consume goods, there is no double coincidence of wants problem and thus a medium of exchange is not essential. Moreover, since there are no informational frictions, agents can settle their trades with any asset or CM goods.

\(^8\)Our results can be generalized to non-separable instantaneous utility functions.

\(^9\)Given our assumptions on the trading opportunities, we do not require linearity in the disutility of labor to guarantee degeneracy of asset holdings across households.
An agent arrives at the beginning of CM with a portfolio of fiat money holdings, nominal and real government bonds, and with a tax liability $T^\text{DM}_t$. This asset position and tax liability reflect the trades in the previous DM. Agents can rebalance their portfolio and have to pay any tax liabilities arising from CM trade before moving into the next DM.\footnote{For sake of generality, we first allow for taxation in the DM market. In what follows it does not matter for the results whether DM taxes are paid in CM or in DM.}

We assume that initial conditions are identical for all agents within the family and across all different households which we later show is consistent with equilibrium. In what follows, let $b^N_{t+1,s}$ and $\{b^i_{t+1,s}\}_{i=1}^{I}$ respectively denote the nominal and real bonds, indexed to the CM consumption good $i$, issued at the end of period $t$ with maturity $s = t + 1, \ldots, \infty$. For nominal bonds, we denote by $q^N_{t,s}$ the price of one dollar in period $s$ in units of dollars in period $t$. Similarly, for real bonds, $q^i_{t,s}$ denotes the price of the CM good $i$ in period $s$ in units of the CM consumption good $i$ in period $t$. We normalize $q^N_{t,t} = 1$ and $q^i_{t,t} = (1 + \tau^C_t)(1 + \tau^X_{i,t})$, where $\tau^C_t$ is a general tax rate on all CM goods and $\tau^X_{i,t}$ represents a specific tax rate for good $i$. From now on we use the consumption good 1 in CM as the benchmark good and set $\tau^X_1 \equiv 0$.\footnote{It is easy to see that this tax structure is equivalent to one tax rate per CM good, which for the one good case it is also equivalent to a labor tax rate.}

As is standard in the literature, we assume that bond payments are not taxable and that governments honor their debt payments. The agent’s budget constraint in CM is then given by

$$\sum_{i=1}^{I} P_t (1 + \tau^C_t)(1 + \tau^X_{i,t})X_{i,t} + m^N_{t+1} + \sum_{s=t+1}^{\infty} q^N_{t,s}b^N_{t+1,s} + P_t \sum_{s=t+1}^{\infty} \sum_{i=1}^{I} q^i_{t,s}b^i_{t+1,s} = P_t w_t L_t - T^\text{DM}_t + m^N_t + \sum_{s=t}^{\infty} q^N_{t,s}b^N_{t,s} + P_t \sum_{s=t}^{\infty} \sum_{i=1}^{I} q^i_{t,s}b^i_{t,s},$$

(2)

where $P_t$ is the nominal price index of CM goods ($X_t$), $m^N_t$ represents fiat money holdings at time $t$, $b^N_t$ denotes the vector of nominal bonds with different maturities, and $b_t$ is the matrix of real bonds for different CM goods and maturities.

The problem of the representative agent consists of maximizing her welfare while taking prices, taxes, initial conditions, and the distribution of money holdings of other agents as given which is summarized by

$$W(m^N_t, b^N_t, b_t) = \max_{X_t, L_t, m^N_{t+1}, b^N_{t+1}} \{U(X_t) - H(L_t) + \beta V(m^N_{t+1}, b^N_{t+1}, b_{t+1}) \text{ subject to (2)}\},$$

where $W(\cdot)$ denotes an agent’s value function at the beginning of the period-$t$ in CM and $V(\cdot)$ represents the family’s value function at the beginning of the period-$t+1$ in DM.
respect to $X_{i,t}$, $m_{t+1}^N$, $b_{t+1,s}^N$ and $b_{t+1,s}^i$ are given by

$$
\frac{\partial}{\partial X_{i,t}} : U_{X_{i,t}} \left( 1 + \tau_{C_t} \right) \left( 1 + \tau_{X_{i,t}} \right) - H_{L_t} = 0,
$$

$$
\frac{\partial}{\partial m_{t+1}^N} : - \frac{U_{X_{i,t}}}{P_t \left( 1 + \tau_{C_t} \right)} + \beta V_{m_{t+1}^N} \left( m_{t+1}^N, b_{t+1}^N, b_{t+1} \right) = 0,
$$

$$
\frac{\partial}{\partial b_{t+1,s}^N} : - \frac{U_{X_{i,t}}}{P_t \left( 1 + \tau_{C_t} \right)} q_{t,s}^N + \beta V_{b_{t+1,s}^N} \left( m_{t+1}^N, b_{t+1}^N, b_{t+1} \right) = 0,
$$

$$
\frac{\partial}{\partial b_{t+1,s}^i} : - \frac{U_{X_{i,t}}}{\left( 1 + \tau_{C_t} \right)} q_{t,s}^i + \beta V_{b_{t+1,s}^i} \left( m_{t+1}^N, b_{t+1}^N, b_{t+1} \right) = 0.
$$

### 2.3.2 The First Stage Problem

After DM opens, buyers and sellers of different families are randomly assigned, with probability $\sigma \in (0, 1)$, to different locations in DM. The matching technology is such that these locations are identical in terms of the measure of buyers and sellers of a given household that are sent to different locations. Thus, as in Shi (1997), members of the same households cannot trade with each other.

Buyers and sellers trade goods for money at competitive prices. The price taking assumption in the search-theoretic models of money is less standard. We choose this DM trading protocol to be closer to the previous literature since in Lucas and Stokey (1983), and subsequent papers, agents trade at competitive markets. As pointed out by Rocheteau and Wright (2005), this “competitive equilibrium” interpretation can be thought as a generalization of Lucas and Prescott (1974) and Alvarez and Veracierto (2000). This competitive environment is still consistent with fiat money being essential as long as trades in DM maintain anonymity, lack of enforcement and there is still a double-coincidence-of-wants problem. Moreover, we do not consider the more standard Nash bargaining protocol in DM as it introduces additional inefficiencies which give additional motives for taxation.\(^{12}\) These additional tax motives are not found in environments based on Lucas and Stokey (1983) so we do not consider them.

Let us now examine the problem of the family in the DM market. The family splits all the fiat currency among its buyers. After buyers and sellers of different families are assigned to their locations, buyers face a cash constraint when making the purchase on behalf of the family. In particular, the total amount of fiat currency given to buyers needs to finance all the purchases that the household makes, that is, $\sum_{i=1}^I \hat{p}_{i,t} x_{i,t} \leq \frac{m_{t}^N}{\kappa}$ where $\hat{p}_{i,t}$ denotes the (normalized) market

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\(^{12}\)See Aruoba, Rocheteau, and Waller (2007) and Gomis-Porqueras and Peralta-Alva (2010) for more on this issue.
price of good $x_{i,t}$. The problem of the household then can be written as follows

$$V(m_t^N, b_t^N, b_t) = \max_{\{x_{i,t},\{l_{i,t}\}} \left\{ \sigma \left[ \kappa \chi_x u(x_t) - (1 - \kappa) \chi_l h \left( \sum_{i=1}^l l_{i,t} \right) + \epsilon_t \left( \frac{m_t^N}{\kappa} - \sum_{i=1}^l \hat{p}_{i,t} x_{i,t} \right) \right] + W \left( m_t^N - \sigma \kappa \sum_{i=1}^l \hat{p}_{i,t} x_{i,t} + \sigma (1 - \kappa) \sum_{i=1}^l \hat{p}_{i,t} (1 - \tau_{i,t}^S) l_{i,t}, b_t^N, b_t \right) \right\},$$

(3)

where $\epsilon_t$ denotes the Lagrange multiplier on the cash constraint, $\tau_{i,t}^S$ is a sales tax rate on DM good $i$, $\sum_{i=1}^l \hat{p}_{i,t} (1 - \tau_{i,t}^S) l_{i,t}$ is the total amount of cash (net of sales taxes) that sellers from the household receive to compensate for their exerted effort, $\chi_x < 1$ and $\chi_l > 1$ represent the loss in utility due to the random assignment process.\(^{13}\) We again use good 1 as the benchmark good and set $\tau_{1,t}^S \equiv 0$. Note that the inflation tax affects all DM goods and therefore the same number of tax instruments are now available in both CM and DM.\(^{14}\) As in Shi (1997), in this environment members of the same household share all of the proceeds as well as the costs of trading in DM before they trade in the next CM.

The first order conditions for consumption and labor are respectively

$$\frac{\partial}{\partial x_{i,t}} : \quad \sigma \kappa \chi_x u_{x_{i,t}} - \sigma \epsilon_t \hat{p}_{i,t} - \sigma \kappa \hat{p}_{i,t} \frac{U_{X_{1,t}}}{P_t(1 + \tau_t^C)} = 0,$$

$$\frac{\partial}{\partial l_{i,t}} : \quad -\sigma (1 - \kappa) \chi_l h_l + \sigma (1 - \kappa) \hat{p}_{i,t} (1 - \tau_{i,t}^S) \frac{U_{X_{1,t}}}{P_t(1 + \tau_t^C)} = 0.$$

Solving for $\epsilon_t$ in the first equation, then the resulting envelope conditions are given by

$$V_{m_t^N} = \sigma \frac{\epsilon_t}{\kappa} + \frac{U_{X_{1,t}}}{P_t(1 + \tau_t^C)} = \sigma \chi_x \frac{u_{x_{i,t}}}{\hat{p}_{i,t}} + (1 - \sigma) \frac{U_{X_{1,t}}}{P_t(1 + \tau_t^C)},$$

.$$V_{b_t^N} = \frac{U_{X_{1,t}}}{P_t(1 + \tau_t^C)} q_{b_t^N},$$

.$$V_{b_t^s} = \frac{U_{X_{1,t}}}{(1 + \tau_t^C)} q_{b_t^s}.$$

### 2.4 Government

We consider a benevolent government that must finance the government spending, $G_t$ in every period, through distortionary taxes and by issuing bonds and fiat money. The real value of all

\(^{13}\)In Shi (1997) the household can choose the search intensity which is costly in utility terms.

\(^{14}\)If we allow for a rate $\tau_{1,t}^S$ different from 0, we have multiple tax structures that decentralize a competitive equilibrium.
issues at every period is assumed to be bounded above by a sufficiently large constant in order to avoid Ponzi schemes. The corresponding government budget constraint is given by

\[
\sum_{i=1}^{I} P_t r_i^C (1 + \tau_{i,t}^C) X_{i,t} + \sum_{i=2}^{I} P_t r_i^X X_{i,t} + T_{t,DM} + m_{t+1}^N + \sum_{s=t+1}^{\infty} q^N_{t,s} b^N_{t+1,s} + P_t \sum_{s=t+1}^{\infty} \sum_{i=1}^{I} q_{t,s}^i b^i_{t+1,s} \geq P_t G_t + m_{t+1}^N + \sum_{s=t+1}^{\infty} q^N_{t,s} b^N_{t+1,s} + P_t \sum_{s=t+1}^{\infty} \sum_{i=1}^{I} q_{t,s}^i b^i_{t+1,s},
\]

where \( T_{t,DM} = \sigma \kappa \sum_{i=2}^{I} \hat{p}_{i,t} \tau_{i,t}^S x_{i,t} \) is the amount of taxes collected in DM.

### 2.5 Market Clearing

To close the model we impose market clearing across the different markets. The aggregate resource constraints in DM and CM imply respectively

\[
\sigma \kappa \sum_{i=1}^{I} x_{i,t} = \sigma (1 - \kappa) l_t,
\]

\[
\sum_{i=1}^{I} X_{i,t} + G_t = L_t.
\]

### 2.6 Monetary equilibrium

Let us denote the return on nominal and real bonds by \( R_{t+1}^N \equiv \frac{q_{t+1,s}^N}{q_{t,s}^N} \) and \( R_{t+1} \equiv \frac{q_{t+1,s}}{q_{t,s}} \) respectively. Given the government policies \( \{ r_i^C, \{ r_i^X \}_{i=2}^{I}, \{ r_i^S \}_{i=2}^{I}, b_{t+1} \}_{t=0}^{\infty}, \) public spending \( \{ G_t \}_{t=0}^{\infty}, \) the initial price level \( P_0 \) and initial conditions \( \{ m_0^N, b_0^N, b_0 \} \), a monetary equilibrium is a collection of sequences \( \{ \{ x_{i,t} \}_{i=1}^{I}, l_t, \{ \hat{p}_{i,t} \}_{i=1}^{I}, \{ X_{i,t} \}_{i=1}^{I}, L_t, R_{t+1}, m_{t+1}^N, P_{t+1}, b_{t+1}^i \}_{t=0}^{\infty} \) satisfying the following

\[
\chi l_t \frac{h_{t}}{\hat{p}_{t,t}} = \frac{U X_{1,t}}{P_t (1 + \tau_t^C)}, \tag{ME1}
\]

\[
\hat{p}_{i,t}(1 - \tau_{i,t}^S) = \hat{p}_{1,t} \text{ for all } i = 2, ..., I, \tag{ME2}
\]

\[
\sigma \kappa \sum_{i=1}^{I} x_{i,t} = \sigma (1 - \kappa) l_t, \tag{ME3}
\]

\[
\sigma \sum_{i=1}^{I} \hat{p}_{i,t} x_{i,t} = \sigma \frac{m_{t+1}^N}{\kappa}, \tag{ME4}
\]
\[
\frac{u_{x_{i,t}}}{\bar{p}_{i,t}} = \frac{u_{x_{1,t}}}{\bar{p}_{1,t}}, \text{ for all } i = 2, ..., I, \quad (\text{ME5})
\]

\[
H_{t} = \frac{U_{X_{i,t}}}{(1 + \tau_{C}^{i})}, \quad (\text{ME6})
\]

\[
\frac{U_{X_{i,t}}}{(1 + \tau_{X}^{i})} = U_{X_{1,t}} \text{ for all } i = 2, ..., I, \quad (\text{ME7})
\]

\[
\sum_{i=1}^{I} X_{i,t} + G_{t} = L_{t}, \quad (\text{ME8})
\]

\[
\frac{U_{X_{1,t}}}{(1 + \tau_{C}^{1})} = \beta \frac{U_{X_{1,t+1}}}{(1 + \tau_{C}^{1+1})} R_{t+1}, \quad (\text{ME9})
\]

\[
\frac{U_{X_{1,t}}}{P_{t}(1 + \tau_{C}^{1})} = \beta \frac{U_{X_{1,t+1}}}{P_{t+1}(1 + \tau_{C}^{1+1})} R_{t+1}^{N}, \quad (\text{ME10})
\]

\[
\frac{U_{X_{1,t}}}{P_{t}(1 + \tau_{C}^{1})} = \beta \frac{U_{X_{1,t+1}}}{P_{t+1}(1 + \tau_{C}^{1+1})} \left( \frac{\chi_{x} u_{x_{1,t+1}}}{\chi_{t} h_{t+1}} \right) + 1 - \sigma \right), \quad (\text{ME11})
\]

\[
\sum_{i=1}^{I} P_{t} \tau_{C}^{i}(1 + \tau_{i,t}^{X}) X_{i,t} + \sum_{i=2}^{I} P_{t} \tau_{i,t}^{X} X_{i,t} + T_{DM}^{N} + m_{t+1}^{N} + \sum_{s=t+1}^{\infty} q_{t,s}^{N} b_{t+1,s}^{N} + P_{t} \sum_{s=t+1}^{\infty} \sum_{i=1}^{I} q_{i,s}^{t} b_{t+1,s}^{i} \geq 0.
\]

\[
\sum_{s=t}^{\infty} q_{i,s}^{N} b_{t,s}^{N} + P_{t} \sum_{s=t}^{\infty} \sum_{i=1}^{I} q_{i,s}^{t} b_{t,s}^{i}, \quad (\text{ME12})
\]

Conditions (ME9)-(ME11) follow from the envelope conditions for fiat money, and by substituting (ME1). By non-arbitrage, we find \( q_{t,s}^{N} = q_{t,s}^{r} q_{r,s}^{i} \) from equation (ME6), and similarly for nominal bonds from (ME10). We also find that \( q_{t,s}^{N} = \frac{P_{t}}{P_{s}} q_{t,s}^{1} \). The nominal interest rate is \( R_{t+1}^{N} - \frac{1}{q_{t+1}} - 1. \) Note that the distribution between nominal, \( b_{t}^{N} \), and real, \( b_{t} \), bonds, the maturity of the bonds, as well as the initial level of prices \( P_{0} \) are chosen by the government.

In this environment, money provides some rents in the sense that, without it, trades in DM simply could not occur, decreasing welfare. This is implicit in the first order condition for money holdings (ME11). A household chooses to hold money with the anticipation of consumption in the next DM. Money would never be held unless the natural surplus in DM is positive, i.e. \( \sigma \left[ \kappa x_{x} u(x_{t}) - (1 - \kappa) \chi_{t} h(l_{t}) \right] \geq 0. \) This surplus is captured in (ME11) by the following condition\(^{15}\)

\[
R_{t+1}^{N} - \frac{1}{q_{t+1}} - 1 = \sigma \left( \frac{\chi_{x} u_{x_{1,t+1}}}{\chi_{t} h_{t+1}} - 1 \right) \geq 0,
\]

implies a zero lower bound on nominal interest rates, which needs to be satisfied for a monetary

\(^{15}\)Nominal interest rates can be written in terms of only allocation by writing it in terms of consumption of DM good 1, as other goods are subjected to sales tax rates.
equilibrium to exist. This bound can be written as follows

$$\chi_x u_{x_{1,t+1}} - \chi_l h_{l_{t+1}} \geq 0; \quad (5)$$

where the Friedman rule is satisfied whenever $$\chi_x u_{x_{1,t+1}} = \chi_l h_{l_{t+1}}$$.

In a monetary equilibrium, the level of prices $$P_t$$ in the CM affects the family choices in DM. More precisely, for given tax rates, (ME6)-(ME8) determine consumption and labor choices in CM which are independent of $$P_t$$. Then, using (ME6), condition (ME1) can be written as

$$\hat{p}_{1,t} = \chi_l \frac{h_{l_t}}{H_{L_t}} P_t,$$

which together with (ME2) implies that a higher level of prices $$P_t$$ induces higher prices $$\hat{p}_{i,t}$$ for all $$i$$ in DM. Then, given $$m_N^t$$, this translates into lower consumption $$x_t$$ through a tighter cash constraint (ME4) and lower effort $$l_t$$ from (ME3). This mechanism linking the price level effect in CM to DM takes place in all periods, including date 0.

3 Optimal Fiscal and Monetary Policy

In this Section we study optimal fiscal and monetary policy using the Ramsey approach. We consider a benevolent government that seeks to maximize the welfare of the representative household, which is given by

$$\sum_{t=0}^{\infty} \beta^t \left( \sigma \kappa \chi_x u(x_t) - \sigma (1 - \kappa) \chi_l h(l_t) + U(X_t) - H(L_t) \right). \quad (6)$$

As is common in the Ramsey literature, we adopt the primal approach and cast the Ramsey problem as that of a government that chooses allocations subject to feasibility while raising an exogenous government revenue, ensuring that the resulting allocations are implementable. In order to this, we first derive the implementability constraint by adding the household’s budget constraints over time and imposing the monetary equilibrium conditions. As shown in the Appendix, we obtain the following implementability condition:

$$\sum_{t=0}^{\infty} \beta^t \left( \sum_{i=1}^{I} U_{X_{i,t}} X_{i,t} - H_{L_t} L_t \right) + \sum_{t=1}^{\infty} \beta^t \left( \sigma \kappa \chi_x \sum_{i=1}^{I} u_{x_{i,t}} x_{i,t} - \sigma (1 - \kappa) \chi_l h_{l_t} l_t \right) =$$

$$\frac{H_{L_0}}{P_0} \left( (1 - \sigma \Psi(x_0)) m_N^0 + \sum_{s=0}^{\infty} g_{0,s} N_{0,s} \right) + \sum_{s=0}^{\infty} \sum_{i=1}^{I} \beta^s U_{X_{i,s}} b_{0,s}^i, \quad (7)$$
As we can see, the implementability condition (7) does not directly capture the DM choices in period 0, it only does so through condition (ME2) which is used in \( \Psi(x_0) \). These choices are summarized as follows
\[
H_{L_0} m_0^N \frac{N_0}{\kappa} = \chi_l h_{l_0} \left( \sum_{i=1}^I \Delta(x_{1,0}, x_{i,0}) x_{i,0} \right),
\]
which is the period 0 cash constraint (ME4) after substituting in DM prices from (ME1) and (ME5), and the tax rate from (ME6). In (8), we define \( \Delta(x_{1,0}, x_{i,0}) \equiv \frac{u_{x_{i,0}}}{u_{x_{1,0}}} \) for all \( i \), if there are taxes available in DM, and 1 otherwise.

Given some initial conditions, the Ramsey problem can be stated as choosing \( \{x_t, l_t, X_t, L_t\} \) and \( P_0 \) to maximize welfare (6) subject to the resource constraints in DM and CM, (ME3) and (ME8), the implementability conditions (7)-(8) and the zero-bound constraint (5). If the government does not have the ability to tax in DM, then the following decentralization constraints, which come from (ME2) and (ME5), must be also added to the government problem:
\[
u_{x_{i,t}} = u_{x_{1,t}}, \text{ for all } i = 2, \ldots, I.
\]

We now construct the Lagrangian of the Ramsey problem in period 0 in a general form that captures the possibilities of having and not having taxes available in DM. Substituting in labor in DM and CM through the resource constraints, (ME3) and (ME8), this Lagrangian can be written as follows
\[
L = \sum_{t=0}^\infty \beta^t \left\{ \sigma \kappa \chi_x u(x_t) - \sigma(1 - \kappa)\chi_l h \left( \frac{\kappa}{1 - \kappa} \sum_{i=1}^I x_{i,t} \right) + U(X_t) - H(\sum_{i=1}^I X_{i,t} + G_t) \right\}
\]
\[
+ \sum_{t=0}^\infty \beta^t \sum_{i=2}^I \sigma \kappa \varphi_{i,t}^0 \left( u_{x_{i,t}} - u_{x_{1,t}} \right) + \sum_{t=0}^\infty \beta^t \sigma \kappa \mu_t^0 \left( \chi_x u_{x_{1,t}} - \chi_l h_{l_t} \right)
\]
\[
+ \lambda^0 \left( \sum_{t=0}^\infty \beta^t \left( \sum_{i=1}^I U_{X_{i,t}} X_{i,t} - H_{L_t} \left( \sum_{i=1}^I X_{i,t} + G_t \right) \right) + \sum_{t=0}^\infty \beta^t \left( \sigma \kappa \chi_x \sum_{i=1}^I u_{x_{i,t}} x_{i,t} - \sigma \kappa \chi_l h_{l_t} \sum_{i=1}^I x_{i,t} \right) \right)
\]
\[
- \lambda^0 \left( \frac{H_{L_0}}{P_0} \left( 1 - \sigma \Psi(x_0) \right) m_0^N + \sum_{s=0}^\infty q_{0,s} b_{0,s}^N \right) + \sum_{s=0}^\infty \sum_{i=1}^I \beta^s U_{X_{i,s}} b_{i,s}^0 \right) \right)
\]
\[
+ \sigma \kappa \eta_0^0 \left( \frac{H_{L_0}}{P_0} \frac{m_0^N}{\kappa} - \chi_l h_{l_0} \left( \sum_{i=1}^I \Delta(x_{1,0}, x_{i,0}) x_{i,0} \right) \right)
\]

where the multiplier on the implementability condition (7) is denoted \( \lambda^0 \). This multiplier is

\[16\] Alternatively, one can include (8) by substituting \( P_0 \) in the implementability condition (7).
strictly positive and measures the cost of distortionary taxation. The multiplier corresponding to condition (8) is denoted \( \eta_0^0 \), which is strictly positive and measures the gains from ensuring a monetary equilibrium in DM in period 0. The multipliers on the decentralization (9) and zero bound constraints (5) are given by \( \varphi^0_{i,t} \) and \( \mu^0_t \), respectively.\(^{17}\) In all previous multipliers, the superscript 0 denotes that the choices are made by the government in period 0. As previously mentioned, this Lagrangian allows for the possibilities of having (or not) taxes in DM. When taxes are available in DM, we set \( \varphi^0_{i,t} = 0 \) and \( \Delta(x_{1,0}, x_{1,0}) = \frac{u_{x_1,0}}{u_{x_1,0}} \). When they are not available, we set \( \Psi(x_0) = 0 \) and \( \Delta(x_{1,0}, x_{1,0}) = 1 \).

Recall that \( q_{0,0}^N = 1 \) and \( q_{0,s}^N = \prod_{k=1}^{s} \frac{1}{1+\varphi^0_{i,t}} = \prod_{k=1}^{s} \frac{1}{\left(\frac{x_{x_1,k}^{u_1^{x_1,k}}}{x_{x_1,k}^{u_1^{x_1,k}}+1}+1-\sigma\right)} \) for \( s \geq 1 \). Then \( q_{0,s}^N \) is a function of solely \( \{(x_{1,t}, ..., x_{I,t})\}_{i,t=1}^{s} \). To keep track of its partial derivatives, we define

\[
\theta_{i,t}^0, s = \frac{1}{\beta t \sigma \kappa x u_{x_1,t} x_{i,t}} \frac{H L_0}{P_0} \frac{\partial q_{0,s}^N}{\partial x_{i,t}} \text{ for all } s \geq 1, \text{ and for all } i = 1, ..., I.
\]

Then, the first order conditions for \( X_{i,t} \) for all \( i \geq 1 \) and \( t \geq 0 \) are given by\(^{18}\)

\[
\frac{\partial}{\partial X_{i,t}} : \quad (1+\lambda^0) U_{X_{i,t}} + \lambda^0 U_{X_{i,t} x_{i,t}} (X_{i,t} - b_{0,t}^0) = (1 + \lambda^0) H_{L_t} + \lambda^0 H_{L_t, L_t, L_t}. \tag{10}
\]

Similarly, the first order conditions for \( x_{i,t} \) for all \( i \geq 1 \) and \( t \geq 1 \) are given by

\[
\frac{\partial}{\partial x_{i,t}} : \quad (1+\lambda^0) x u_{x_1,t} x_{i,t} + \lambda^0 x u_{x_1,t} x_{i,t} \left( x_{i,t} - \sum_{s=t}^{\infty} \theta_{i,s}^0, b_{0,s}^0 \right) + \Sigma^0_{i,t} = (1 + \lambda^0) \chi h_{i,t} + \lambda^0 \chi h_{i,t, i,t} L_t. \tag{11}
\]

which in period 0 become

\[
\frac{\partial}{\partial x_{i,0}} : \quad \chi u_{x_1,0} + \Sigma^0_{i,t} + \lambda^0 \frac{H L_0}{P_0} m_0^N \frac{1}{\kappa} \Psi_{x_{i,0}},
\]

\[
= \left( 1 + \eta_0^0 \right) \left( \Delta(x_{1,0}, x_{1,0}) + \sum_{j=1}^{I} \Delta_{x_{i,0}} x_{j,0} \right) \chi h_{i,0} + \frac{\eta_0^0 \chi h_{i,0,l_0} l_0 \frac{1}{\kappa}}{1 - \kappa} \left( \sum_{i=1}^{I} \Delta(x_{1,0}, x_{1,0}) x_{i,0} \right), \tag{12}
\]

with

\[
\Sigma^0_{i,t} = \left\{ - \sum_{i=2}^{N} \varphi^0_{i,t} u_{x_1,t, x_{1,t}} + \mu^0_t \left( x u_{x_1,t, x_{1,t}} - \chi l \frac{1}{1-\kappa} h_{i,t, l_t} \right) \text{ for } i = 1 \right\},
\]

\[
\varphi^0_{i,t} u_{x_1,t, x_{1,t}} - \mu^0_t \chi l \frac{1}{1-\kappa} h_{i,t, l_t} \text{ for all } i \geq 2.
\]

\(^{17}\)The multipliers on (5), (8) and (9) are multiplied by \( \sigma \kappa \) to economize on notation.

\(^{18}\)The first order condition in period 0 already includes equation (13).
Finally, the first order condition for the initial level of prices \( P_0 \) is given by

\[
\sigma \eta_0^0 N_0^0 = \lambda_0 \left( 1 - \sigma \Psi(x_0) \right) m_0^N + \sum_{s=0}^{\infty} q_{0,s}^N b_{0,s}^N
\]

(13)

where the left-hand side measures the marginal cost from extra inflation at 0, as it reduces the gains from the period 0 monetary equilibrium by decreasing consumption in DM. The right-hand side measures the marginal gain from that extra inflation at 0, as it reduces the need for distortionary taxation by lowering the real value of the nominal liabilities. Equation (13) is very similar to condition (25) found in Persson et al. (2006).

In the cash-in-advance model of Lucas and Stokey (1983), the optimal initial level of prices is infinity. Our choice of timing is consistent with the assumption of beginning-of-period money balances as in Nicolini (1998) and in Persson et al. (2006). Then it is easy to see that the optimal initial level of prices is always finite.\(^{19}\) In our monetary economy, the choice of the price level at time 0 must balance the motive to lower the distortionary cost of taxation versus the reduction in consumption in DM, \( x_0 \), as seen in (13). Very high inflation drastically reduces consumption in DM, and given our assumed utility function (that \( u_{x_i} = \infty \) whenever \( x_i = 0 \)), this can never be optimal even in at period 0. This also suggests that the time inconsistency problem of monetary policy in our environment is not as severe.

We now characterize the Ramsey policy plan. As usual, we assume that there exists an optimal solution to the Ramsey problem. This solution can be decentralized with the instruments \( \{ \tau_t^C, \{ \tau_{i,t}^X \}_{i=2}, R_t^N, \{ \tau_{i,t}^S \}_{i=2} \} \), in any period \( t \geq 0 \). By combining the consumption-labor decisions of the agent in the CM, equations (ME6)-(ME7), and the first order conditions of \( X_{i,t} \) for the government, we obtain the following Ramsey consumption tax rates from period 0 onwards:

\[
1 + \tau_t^C = 1 + \frac{\lambda_0}{1 + \lambda_0} \left\{ \frac{H_{t,t} L_t L_t}{H_{t,t}} - \frac{U_{X_{1,t}X_{1,t}}}{H_{t,t}} (X_{1,t} - b_{0,t}) \right\}, \tag{14}
\]

\[
(1 + \tau_{i,t}^X)(1 + \tau_t^C) = 1 + \frac{\lambda_0}{1 + \lambda_0} \left\{ \frac{H_{t,t} L_t L_t}{H_{t,t}} - \frac{U_{X_{1,t}X_{1,t}}}{H_{t,t}} (X_{i,t} - b_{0,t}) \right\}. \tag{15}
\]

As we can see, the level of taxes increases with the need of distortionary taxation \( \lambda_0 \). Moreover, assuming zero initial bonds and constant elasticities of intertemporal substitution for consumption and labor, the optimal consumption tax rates are constant over time.

We next examine whether the Friedman rule is optimal. Combining the first order condition of the government for \( x_{1,t} \) with the monetary equilibrium conditions, and assuming that the decentralization constraints are not binding (9), we find that the optimal \( R_t^N \) for \( t \geq 1 \) is given

\(^{19}\)If the timing was such that the DM occurs after CM, then the optimal \( P_0 \) would be infinite.
by \(^{20}\)

\[
R_t^N = 1 + \sigma \frac{\lambda_0}{1 + \lambda_0} \left\{ \frac{h_{t,i,t} l_{t}}{h_{t,i}} - \frac{\chi_x}{\chi_t} \frac{u_{x_{1,t},x_{1,t}}}{h_{t,i}} \left( x_{1,t} - \sum_{s=t}^{\infty} \theta_0^{N} b_{0,s}^{N} \right) \right\},
\]

(14)

which, for zero initial nominal bonds maturing after period 1, implies a nominal interest rate, \(R_t^N - 1\), that is strictly positive and increasing with the need of distortionary taxation \(\lambda_0\). Therefore, the Friedman rule is not optimal in our search-theoretical economy, even at steady state. This result is consistent with Aruoba and Chugh (2010) who consider an environment where the government can commit to future policies. As opposed to these authors, our interpretation for the Friedman rule not being optimal relates to the number of tax instruments available, as also discussed in Kocherlakota (2005). A general consumption tax in DM, similar to \(\tau_t^C\) in CM, could replace the inflation tax and nominal interest rates could be set to zero.

As for period 0, consider that taxes are available and optimally uniform in DM, then the first order condition for \(x_{i,0}\) can be written as

\[
\chi_x u_{x_{i,0}} = (1 + \eta_0^0) \chi_t h_{t,0} + \eta_0^0 \chi_t h_{t,0} l_{0},
\]

(15)

which, after substituting \(\eta_0^0\) from (13) we get

\[
R_0^N = 1 + \lambda_0 \left( \frac{m_0^N + \sum_{s=0}^{\infty} q_0^N b_{0,s}^N}{m_0^N} \right) \left( 1 + \frac{h_{t,0} l_{0}}{h_{t,0} l_{0}} \right).
\]

Therefore the optimal nominal interest rate in period 0 is also strictly positive and increases with the cost of distortionary taxation \(\lambda_0\) as well as the amount of initial nominal bonds relative to money holdings. Note also that from (15), \(\eta_0^0\) can be written as an increasing function of the initial optimal nominal interest rate.

If sales taxes are available in DM, the optimal sales tax rate on good \(i\) for period \(t \geq 1\) can be similarly obtained from

\[
\frac{R_t^N}{1 - \tau_{t,i,t}^S} = \sigma + \sigma \frac{\lambda_0}{1 + \lambda_0} \left\{ \frac{h_{t,i,t} l_{t}}{h_{t,i}} - \frac{\chi_x}{\chi_t} \frac{u_{x_{1,t},x_{1,t}}}{h_{t,i}} \left( x_{1,t} - \sum_{s=t}^{\infty} \theta_0^{N} b_{0,s}^{N} \right) \right\} + (1 - \sigma) \frac{u_{x_{1,t}}}{u_{x_{1,t}}}.
\]

If the utility function \(u(\cdot)\) is homothetic, then uniform commodity taxation in DM is optimal. Then all DM goods would be subject to the inflation tax implied by (14) and sales tax rates would be set to zero, \(\tau_{t,i,t}^S = 0\) for all \(i \geq 2\).

In the next Section, we study whether the Ramsey policy plan, that we just characterized, is time consistent.

\(^{20}\)When taxes in DM are not available and the decentralization constraints (9) bind, the optimal level of nominal interest rates will be affected by the additional relative motives for taxation, just as in Correia (1996)’s model of incomplete taxation.
4 Time Consistency of the Optimal Policy

As usual, the structure of government bonds is irrelevant in the economy with commitment. In other words, the real value of total government liabilities is uniquely determined by the Ramsey allocation, but the composition and maturity of these obligations are not. In this Section, we explore whether there is an optimal structure of real and nominal government bonds that makes the optimal fiscal and monetary policy time consistent.

In our search-theoretic economy, we find the following result.

**Proposition 1** Optimal fiscal and monetary policies are in general time inconsistent.

Proof: See Appendix.

Proposition 1 emphasizes the fact that in this environment we have a limited debt structure and therefore not enough instruments to guarantee time consistency. Following Tinbergen (1956)’s reasoning, a government requires at least as many debt instruments as policies to induce the next government to not deviate and to optimally continue with the Ramsey policy plan previously selected. Taking further Tinbergen’s arguments, we find that time consistency can be imposed in some circumstances as stated in the next proposition.

**Proposition 2** There exists an optimal structure of real and nominal government bonds that makes the optimal fiscal and monetary policies time consistent whenever

(i) taxes in DM are not available;
(ii) taxes in DM are available and satisfy the conditions for optimal uniform taxation.

Proof: See Appendix.

As we can see from Proposition 1 and 2, there are two important cases where time consistency can be restored. First, if taxes in the decentralized anonymous markets are not available, the multipliers on the decentralization constraints, which are not determined by the allocation, can be utilized as additional instruments to ensure time consistency. Second, if taxes in the decentralized markets are available, time consistency arises when the different decentralized market goods satisfy the conditions necessary for optimal uniform commodity taxation. However, if the government finds optimal to use non-uniform taxation across goods in decentralized markets and has access to such taxation in these markets, then optimal fiscal and monetary policies are time inconsistent.

\[21\] A similar result is found in a capital taxation problem by Domínguez (2007). The author finds that in problems where a unique debt structure guarantees time consistency, multiplicity of optimal debt structures arise if there are additional constraints on the implementable allocation that introduce extra multipliers in the government problem. For example, imposing that capital tax rates cannot be above 100 %, if binding, generates multiplicity.
When time consistency is restored, the optimal debt structure is very similar to the one found by Persson et al. (2006). Eliminating the incentives to increase the initial level of prices implies an overall amount of nominal bonds issues relative to money holdings for which initial net nominal liabilities are positive. Eliminating the incentives to deviate from the previously chosen DM taxes determines the maturity of the nominal bonds.

In our economy the Friedman rule is not optimal. We next examine situations where the Friedman rule can be optimal and re-examine the results of Alvarez et al. (2004). Consider instantaneous utility functions \(u(\cdot)\) and \(h(\cdot)\) that solve the Ramsey problem and that satisfy the optimal Friedman rule \(\chi_x u_{x_1,t} = \chi_l h_{l,t}\). Here we are thinking of allowing for a positive satiation level for good 1, \(x_1\), so that \(u(\cdot)\) would not satisfy monotonicity neither concavity for good 1 while maintaining all previous assumptions regarding all other decentralized markets goods. Then, it is easy to see, that our monetary economy is still not identical to a real economy. Even if nominal interest rates are optimally zero, the government still faces a taxation problem in decentralized markets. And, if taxes in decentralized markets are available and optimally different across goods, then optimal fiscal and monetary policies are time inconsistent. This time inconsistency is a direct consequence of the inability to influence the fiscal choices in decentralized markets. Then the results of Alvarez et al. (2004), that ensure that optimal policies are time consistent when the Friedman is satisfied, are not applicable in this environment.

5 Conclusions

This paper has investigated the time consistency of optimal monetary and fiscal policies in economies with multiple consumer goods. We have shown that when there are many government policy decisions per period affecting nominal variables, optimal monetary and fiscal policies are in general time inconsistent. The main reason behind this result is that there are insufficient instruments to influence future governments’ decisions. The insufficiency of debt instruments is a necessary condition for the existence of money. If governments could issue bonds that pay the right cash goods at the right time/location, fiat money would not be essential.

Furthermore, we find that the frictions that make money essential in our economy are also relevant for time consistency. If the anonymity and lack of enforcement in decentralized markets are such that preclude governments from taxing agents in these markets, then optimal monetary and fiscal policies are time consistent. If, on the other hand, the government can enforce taxation on anonymous individuals (by, for example, taxing the firms where they work in these decentralized markets), then optimal policies will be time inconsistent when the different decentralized markets goods require an optimal non-uniform tax. These results are a direct consequence of how fiat money is modeled which has important implications for other parts of the model as emphasized by Wallace (2001). The framework in this paper highlights how informational frictions
that make fiat money essential affect the debt and tax instruments available which in turn affect
the time consistency of optimal fiscal and monetary policies.

We believe that our results are an example of a broader issue. The nominal submarket with
multiple goods and informational frictions imposes restrictions on the feasible debt instruments,
and nominal bonds can only influence one aspect of the nominal basket but not all its compo-
nents in a different way. In this paper, we have adopted a large family approach, where all the
randomness of the location assignment are smoothed out, and have ignored the possibility of a
random matching process leading to different ex-post realizations. If we had included these addi-
tional frictions in the environment, the additional information from one period to the next would
provide an additional motive for future governments to deviate from previous announcements.

References

Securities,” *Journal of Monetary Economics*, 37, 397-419.


*Journal of Monetary Economics*, 54, 2636-2655.


Public Economics*, 60, 147-151.

Monetary Economics*, 54, 686-705.

in a Search Theoretic Model of Monetary Exchange,” *European Economic Review*, 54(3),
331-344.


Appendix

The Implementability Condition

The buyer’s consumption is determined by the cash constraint (ME4) which, multiplied by $\beta_t x^u_{x,t} \hat{p}_{1,t}$ and using (ME5), becomes

$$\sigma \kappa \sum_{i=1}^{I} \beta_t x^u_{x,t} x_{i,t} = \beta_t \sigma x^u_{x,t} \frac{m_t^N}{\hat{p}_{1,t}}.$$  \hspace{1cm} (16)

The seller’s decision is given by the first order condition (ME1), which multiplied by $-\beta_t \hat{p}_{1,t} l_t$, and using (ME2)-(ME4), can be written as

$$-\beta_t \sigma (1-\kappa) x^l_{x} h_t l_t = -\beta_t \sigma m_t^N - T_t^{DM} \frac{U_{X_{1,t}}}{P_t (1 + \tau_t^C)}.$$ \hspace{1cm} (17)

We next write $T_t^{DM} = \sigma \kappa \sum_{i=2}^{I} \hat{p}_{i,t} \tau_{i,t}^S x_{i,t} l_t$ in terms of allocation. From (ME2) and (ME5), we obtain $\hat{p}_{i,t} = \hat{p}_{1,t} u_{x_{i,t}}^u$ and $\tau_{i,t}^S = 1 - \frac{u_{x_{i,t}}^u}{u_{x_{1,t}}^u}$. From (16), we get $\hat{p}_{1,t} = \frac{u_{x_{1,t}}^u}{\sum_{i=1}^{I} u_{x_{i,t}} x_{i,t}^u} m_t^N$. Therefore, we find

$$T_t^{DM} = \sigma \Psi(x_t) m_t^N,$$ \hspace{1cm} (18)

where $\Psi(x_t) \equiv \sum_{i=2}^{I} \left( \frac{u_{x_{i,t}} - u_{x_{1,t}}}{\sum_{i=1}^{I} u_{x_{i,t}} x_{i,t}^u} \right).$

Adding equations (16)-(17), we summarize DM in

$$\beta^t \left( \sigma \kappa x^u_{x} \sum_{i=1}^{I} u_{x_{i,t}} x_{i,t} l_t - \sigma (1-\kappa) x^l_{x} h_t l_t \right) = \beta^t \left( \sigma \left( \frac{x^u_{x}}{x^l_{x}} h_t - 1 \right) + \sigma \Psi(x_t) \right) \frac{m_t^N}{P_t (1 + \tau_t^C)} U_{X_{1,t}}.$$ \hspace{1cm} (18)

This equation, (18), in period 0 simplifies to (8).

After DM, given our structure, the household’s money holdings are $m_t^N - T_t^{DM}$. Multiplying the budget constraint (2) by $\beta^t \frac{U_{X_{1,t}}}{P_t (1 + \tau_t^C)}$, and using conditions (ME6) and (ME7), we get

$$\beta^t \left( \sum_{i=1}^{I} U_{X_{i,t}} X_{i,t} - H_{L_t} L_t \right) =$$

$$\beta^t \frac{U_{X_{1,t}}}{P_t (1 + \tau_t^C)} \left( m_t^N - \sigma \Psi(x_t) m_t^N - m_{t+1}^N + \sum_{s=t}^{\infty} q_{t,s}^N b_{t,s}^N - \sum_{s=t+1}^{\infty} q_{t,s}^N b_{t+1,s}^N + \sum_{s=t}^{\infty} I I_{i,s} b_{i,s}^i - \sum_{s=t+1}^{\infty} I I_{i,s} b_{i+1,s}^i \right).$$ \hspace{1cm} (19)

As CM opens in period 0 after DM, we add equation (19) from $t \geq 0$ and condition (18) from
period 1 onwards, to obtain

\[
\sum_{t=0}^{\infty} \beta^t \left( \sum_{i=1}^{I} U_{X_{i,t},X_{i,t}} - H_{L_t} L_t \right) + \sum_{t=1}^{\infty} \beta^t \left( \sigma \kappa \chi_x \sum_{i=1}^{I} u_{x_{i,t},x_{i,t}} - \sigma(1 - \kappa) \chi \mu_{H_{t},L_t} \right) =
\]

\[(1 - \sigma \Psi(x_0)) m_0^N \frac{U_{X_{1,0},X_{1,0}}}{P_0(1 + \tau_{0}^C)} + \sum_{t=1}^{\infty} \beta^{t-1} \left( \beta \left( \frac{\chi_x u_{x_{1,t},X_{1,t}}}{H_{t}} + 1 - \sigma \right) \frac{U_{X_{1,t},X_{1,t}}}{P_t(1 + \tau_{t}^C)} - \frac{U_{X_{1,t-1},X_{1,t-1}}}{P_{t-1}(1 + \tau_{t-1}^C)} \right) m_t^N +
\]

\[
\sum_{t=0}^{\infty} \beta^t \frac{U_{X_{1,t}}}{P_t(1 + \tau_{t}^C)} \left( \sum_{s=t}^{\infty} q_{s,t}^N b_{s,t}^N - \sum_{s=t+1}^{\infty} q_{s,t} b_{s,t+1,s} + \sum_{s=t}^{\infty} \sum_{i=1}^{I} q_{i,s}^i b_{i,s}^i - \sum_{s=t+1}^{\infty} \sum_{i=1}^{I} q_{i,s}^i b_{i,s+1,s} \right). \tag{20}
\]

Imposing the necessary conditions (ME6)-(ME8), the RHS of (20) becomes

\[
\frac{H_{L_0}}{P_0} \left( (1 - \sigma \Psi(x_0)) m_0^N + \sum_{s=0}^{\infty} q_{0,s}^N b_{0,s}^N + P_0 \sum_{s=0}^{\infty} \sum_{i=1}^{I} q_{i,s}^i b_{i,s}^i \right).
\]

Finally, using \( q_{0,s}^i = \beta^s U_{X_{i,s},X_{i,s}} \frac{1}{U_{X_{i,0}}}, \) the implementability condition becomes

\[
\sum_{t=0}^{\infty} \beta^t \left( \sum_{i=1}^{I} U_{X_{i,t},X_{i,t}} - H_{L_t} L_t \right) + \sum_{t=1}^{\infty} \beta^t \left( \sigma \kappa \chi_x \sum_{i=1}^{I} u_{x_{i,t},x_{i,t}} - \sigma(1 - \kappa) \chi \mu_{H_{t},L_t} \right) =
\]

\[
\frac{H_{L_0}}{P_0} \left( (1 - \sigma \Psi(x_0)) m_0^N + \sum_{s=0}^{\infty} q_{0,s}^N b_{0,s}^N \right) + \sum_{s=0}^{\infty} \sum_{i=1}^{I} \beta^s U_{X_{i,s},b_{i,s}^0}.
\]

**Proof of Propositions 1 and 2.**

Let us first present the Ramsey allocations chosen by the government at 0 and the government at 1. The Ramsey allocation evaluated at date 0 for all \( t \geq 1 \) is summarized in:

\[
(1 + \lambda^0) U_{X_{1,t}} + \lambda^0 U_{X_{1,t},X_{1,t}} (X_{1,t} - b_{0,t}^0) = (1 + \lambda^0) H_{L_t} + \lambda^0 H_{L_{t},L_t} L_t, \tag{21}
\]

\[
(1 + \lambda^0) \chi_x u_{x_{1,t}} + \lambda^0 \chi_x u_{x_{1,t},x_{1,t}} \left( x_{i,t} - \sum_{s=t}^{\infty} \theta_{0,s}^i b_{0,s}^i \right) + \sum_{i=0}^{\infty} = (1 + \lambda^0) \chi \mu_{H_{t},L_t} + \lambda^0 \chi \mu_{H_{t},L_{t},L_t} L_t, \tag{22}
\]

together with the resource constraints, (ME3) and (ME8), the implementability constraint (7) and, if taxes in DM are not available, the decentralization constraints (9).

The Ramsey allocation evaluated at date 1 is summarized in:

\[
(1 + \lambda^1) U_{X_{1,t}} + \lambda^1 U_{X_{1,t},X_{1,t}} (X_{1,t} - b_{1,t}^1) = (1 + \lambda^1) H_{L_t} + \lambda^1 H_{L_{t},L_t} L_t, \forall t \geq 1 \tag{23}
\]
\[(1 + \lambda^1) \chi_x u_{x_{1}, t} + \lambda^1 \chi_x u_{x_{1, t}, x_{t}, t} \left(x_{i, t} - \sum_{s=t}^{\infty} \theta_{i, t}^{s} h_{t}^{N_{s}}\right) + \Sigma_{i, t}^{1} = (1 + \lambda^1) \chi_l h_{t} + \lambda^1 \chi_l h_{t, l, t}, \quad \forall t \geq 2\]

which in period 1 becomes
\[
\chi_x u_{x_{1}, t} + \Sigma_{i, t}^{1} + \lambda^1 \frac{H_{L_1}}{P_1} m_{1}^{N} \frac{1}{\kappa} \Psi_{x_{1}, t} = \left(1 + \eta_{1}^1 \left(\Delta(x_{1,1}, x_{1,1}) + \sum_{j=1}^{I} \Delta_{x_{1}, x_{j,1}}\right)\right) \chi_l h_{t} + \eta_{1}^1 \chi_l h_{t, l, t} \frac{\kappa}{1 - \kappa} \left(\sum_{i=1}^{I} \Delta(x_{1,1}, x_{i,1}) x_{i,1}\right),
\]

where \(\eta_{1}^1\) satisfies
\[
\sigma \eta_{1}^1 m_{1}^{N} = \lambda^1 \left(1 - \sigma \Psi(x_{1})\right) m_{1}^{N} + \sum_{s=0}^{\infty} q_{1,s}^{N} b_{1,s}^{N},
\]

together with the resource constraints, (ME3) and (ME8), the implementability constraints (7) and (8) evaluated in period 1, and, if necessary, the decentralization constraints (9).

We now explore whether the government at date 0 can design a debt structure that eliminates the incentives to deviate, by the government at date 1, from the Ramsey plan chosen at date 0. To do this, we first assume that the Ramsey allocation evaluated in period 0 solves also the Ramsey problem in period 1 and then we check whether the existing debt instruments are sufficient to guarantee that. Note that the resource constraints, (ME3) and (ME8), and decentralization constraints (9) are the same in both plans and therefore do not pose any additional incentives to deviate. The initial level of prices is a function of the first period allocation, equation (8) evaluated in period 1, and will be satisfied as long as we guarantee the same allocation.

First, we analyze the CM choices, \(X_{i, t}\) for all \(t \geq 1\), by subtracting (21) from (23) and solving for the real bonds inherited by the government at date 1:

\[
b_{i, t}^1 = \left(\frac{\lambda^0}{\lambda^1}\right) b_{i, t}^0 + \left(1 - \frac{\lambda^0}{\lambda^1}\right) \left(X_{i, t} + \frac{U_{X_{i, t}}}{U_{X_{i, t}, X_{i, t}} - \frac{H_{L_1} + H_{L_1, l, L}}{U_{X_{i, t}, X_{i, t}}}}\right).
\]

Next, we study the first period DM choices, \(x_{i,1}\), by subtracting (22) at \(t = 1\) from (25), substituting the value of \(\eta_{1}^1\) from the condition for \(P_1\) by the government at 1, (26), which yields

\[
\left(\frac{(1 - \sigma \Psi(x_{1})) m_{1}^{N} + \sum_{s=0}^{\infty} q_{1,s}^{N} b_{1,s}^{N}}{m_{1}^{N}}\right) - \sigma \frac{H_{L_1}}{P_1} m_{1}^{N} \frac{1}{\kappa} \Psi_{x_{1}, t} = \left(\frac{\Sigma_{i,1}^{1} - \Sigma_{i,1}^{0}}{\lambda^1}\right) \frac{\sigma}{\Omega(x_{1})} \left(\chi_l (h_{t1} + h_{t1,l1}) - \chi_x \left(u_{x_{1},1} + u_{x_{1},x_{i,1}} \left(x_{i,1} - \sum_{s=1}^{\infty} \theta_{i,1}^{s} h_{t}^{N_{s}}\right)\right)\right),
\]

where we denote \(\Omega(x_{1}) = \chi_l h_{t1} \left\{\Delta(x_{1,1}, x_{1,1}) + \sum_{j=1}^{I} \Delta_{x_{1}, x_{j,1}} x_{j,1} + \sum_{i=1}^{I} \frac{\kappa}{1 - \kappa} \frac{\sum_{j=1}^{I} \Delta_{x_{1,1}, x_{i,1}} x_{i,1}}{l_i} h_{t1,l1}\right\} \).
Then, we focus on the DM choices, $x_{i,t}$ for all $t \geq 2$, by subtracting (22) from (24) and solving for the nominal bonds inherited by the government at date 1, which are as follows:

$$\sum_{s=t}^{\infty} \theta_{1,s}^{i,t} b_{1,s}^{N} - \left( \frac{\Sigma_{i,1}^{1} - \Sigma_{i,1}^{0}}{\lambda^{1}} \right) \frac{1}{x_{x}, u_{x_{t}, x_{i,t}}} = \left( \frac{\lambda^{0}}{\lambda^{1}} \right)^{\infty} \sum_{s=t}^{\infty} \theta_{0,s}^{i,t} b_{0,s}^{N} + \left( 1 - \frac{\lambda^{0}}{\lambda^{1}} \right) \left( x_{i,t} + \frac{u_{x_{i,t}}}{u_{x_{i,t}, x_{i,t}}} - \frac{\chi_{t}}{\chi_{x}} \left( h_{t} + h_{t,t,l,t} l_{t} \right) \right). \tag{29}$$

In the above, we have presumed that the same Ramsey allocation from period 1 onwards solves the policy plans evaluated at date 0 and at date 1. If there were a solution to the above system of bonds (27)-(29), these equations can be imposed onto the implementability condition (7) for the government in period 1 and would provide the value of $\lambda^{1}$. Plugging back this value of $\lambda^{1}$, the equations (27)-(29) would be the optimal real and nominal bond structure that would induce the government in period 1 to follow the Ramsey policy plan evaluated by the government at date 0. By induction, the same it is true for all later periods, and optimal fiscal and monetary policies would be time consistent.

Now the question is under which conditions there exists a solution to the above system of bonds (27)-(29) for all $i = 1, \ldots, I$ and all $t \geq 1$. First, for real bonds (27), we have as many real bonds $I$ as CM consumption-labor decisions $I$ in each period. Therefore, the real economy counts with the exact number of instruments required for time consistency. Second, for nominal bonds, (28) and (29), we have only one nominal bond $b_{t}^{N}$ per period for all DM consumption-labor decisions. If the number of DM consumer goods would be one, the nominal economy would count with the required number of instruments. However, if the number of DM consumer goods is $I > 1$, we have in principle less instruments than those necessary.

If tax rates in DM are not available, then the decentralization constraints (9) imply

$$\Sigma_{i,t}^{1} = \left\{ \begin{array}{ll}
- \sum_{i=2}^{N} \varphi_{1,t}^{i} u_{x_{i,t}, x_{i,t}} & \text{for } i = 1 \\
\varphi_{1,t}^{i} u_{x_{i,t}, x_{i,t}} & \text{for all } i \geq 2
\end{array} \right\}. $$

The multipliers $\varphi_{1,t}^{i}$ are not determined by the Ramsey allocation. For a given period $t \geq 2$, one can pick the new issues of nominal bonds that solves (29) for good 1 and the multiplier $\varphi_{1,t}^{1}$ that solves (29) for all other $i \geq 2$. Similarly for period 1, but solving (28). Plugging them back together with the real bonds (27) into the implementability condition for the government at 1, this implies a multiplier $\lambda^{1}$ that exactly delivers those decentralization multipliers $\varphi_{1,t}^{i}$ and the same allocation and policy as the ones chosen by the government at period 0.

If tax rates in DM are available, then $\Sigma_{i,1}^{1} = \Sigma_{i,1}^{0} = 0$. Note also that the value of $u_{x_{i,t}, x_{i,t}} \theta_{1,s}^{i,t}$ is determined solely by the nominal interest rate $R_{i+1}^{N} - 1 = \sigma \left( \frac{\chi_{x}}{\lambda^{l}} \frac{u_{x_{i+1,t}}}{h_{t+1}} \right) - 1$ and the Ramsey
allocation, just as $U_{X_{i,t}, X_{i,t}}$ for real bonds (27). Then, in general, there is no one $b^N_{t}$ that satisfies (28) and (29) for all DM goods $i = 1, \ldots, I$. If there were one DM good, equation (28) would determine the total amount of new nominal bonds (relative to money holdings). This amount is not good specific and cannot adjust to the different requirements of each DM good $i$ as the right-hand side of the equation requires. If there were one DM good, equation (29) would determine the new issues of nominal bonds for each maturity $s \geq 1$. This amount is again not good specific as the right-hand side of the equation requires. Therefore, optimal monetary and fiscal policies are, in general, time inconsistent.

However, when sales tax rates in DM satisfy the conditions required for optimal uniform commodity taxation, i.e. $u(\cdot)$ is homothetic, then $u_{x_{i,t}} = u_{x_{1,t}}$ for all $i \geq 2$ in each period and nominal interest rates $R^N_{t+1} - 1$, can now be written as a function of the consumption of each DM good:

$$R^N_{t+1} - 1 = \sigma \left( \frac{X_x u_{x_{1,t+1}}}{X^I h_{t+1}} - 1 \right) = \sigma \left( \frac{X_x (\phi_{1,t+1} u_{x_{1,t+1}} + \ldots + \phi_{I,t+1} u_{x_{I,t+1}})}{h_{t+1}} - 1 \right),$$

where $\sum_{i=1}^{I} \phi_{i,t+1} = 1$. Note that without optimal uniform commodity taxation, $R^N_{t+1}$ can only be written as a function of the allocation when it is written in terms of the consumption of DM good 1; otherwise sales tax rates would appear in the above equation. Then the government in period 0 can now choose each $\phi_{i,t}$ that implies a $\theta_{i,t}$, which solves (28) and (29) for all $i \geq 2$ and let the total amount of new issues of nominal bonds solve (28) for $i = 1$ and the new issues at each maturity solve (29) also for $i = 1$. By doing this, together with the real bonds (27), the government at 0 can provide the right incentives for the government at 1 to continue with the Ramsey plan evaluated at 0. Then, optimal monetary and fiscal policies are time consistent. Alternatively, this can be understood by looking at the Ramsey nominal interest rate and noticing that optimal uniform taxation requires zero sales taxes for all $i \geq 2$ and therefore only one decision per period (not per DM good) needs to be influenced by the government in period 0.

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22Taking into account this possibility, when taxes are not available in DM, there would be multiple debt structures that solve the time inconsistency problem.